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ABSTRACT

This set of five papers is related to the Mathemagenic Activities Program (MAP) for early childhood education of the University of Georgia's Follow Through Program. The MAP is based on Piagetian theory and provides sequentially structured sets of curriculum materials and processes that are designed to continually challenge children to learn about the physical and social environments. Topics discussed in these collected papers focus on the constructivist epistemological basis of the MAP. A constructivist, in a philosophical sense, is one who believes that all knowledge is constructed by the individual. Essay topics include the following: an introduction to radical constructivism: radical constructivism and Piaget's concepts of knowledge: a constructivist model for instruction: and constructivism and educational practices. Additional essays are collected in a volume titled, Epistemology and Education: The Implications of Radical Constructivism for Knowledge Acquisition. The essays discuss the following topics: Piaget and radical constructivist epistemology: the constructivist epistemology in John Dewey, Jean Piaget, and cognitive developmental psychology: epistemology and mathematical proof: applied epistemology: radical constructivism in psychological explanation: constructivism and the curriculum: and constructivism and principles for instruction. (Author/RH)

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MATHEMAGENIC ACTIVITIES PROGRAM: REPORTS ON CONSTRUCTIVISM.
Charles D. Smock, Editor
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Epistemology and Education

The
Implications of Radical Constructivism
for
Knowledge Acquisition

Charles D. Smock, Ernst von Glasersfeld,
Editors

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Preface

Education periodically has to face one kind of crisis or another. Too often, these crises arouse citizens and professionals alike to come up with temporary remedies and pseudo-solutions. Recurring problems and issues should be a warning that some fundamental inconsistencies are hidden behind our current beliefs and prejudices about the educational process.

One issue, generally ignored in education, is the relation between our personal theories about child development and instructional methods, on the one hand, and the relation between the nature of the instructional process and its outcomes, in terms of the type of children "produced," on the other. Thus, a theory of instruction must start with analyses of the psychological development of the child and not with strategies for teaching. Such an analysis, however, leads inevitably to an even more fundamental problem - what is the nature of knowledge and the process of its acquisition?

This is a complex problem and one that is often avoided. But, until educators confront it, we will be offered solutions to the wrong problems. The essays in this collection represent some tentative suggestions for answering the questions about the nature of knowledge and knowledge acquisition. If these papers stimulate others to think about these questions *vis-a-vis* educational issues then we have accomplished our objective.

C. D. Smock
E. von Glasersfeld

Acknowledgements

The personal and intellectual stimulation that led to this collection of essays came, of course, from many people. We feel a very special gratitude to those few we are able to mention here.

Interest in the general constructivistic philosophy emerged in the thinking of C. D. Smock through the process of developing a "shared environment" with Bette Stum Smock. George G. Thompson provided the opportunities and guidance that led to the study of Piagetian ideas. And, Piaget, of course, provided the conceptual keys to the implications of constructivism for education. His deepest appreciation goes to each of these friends and colleagues for supporting his efforts to "see" the relation of epistemology and education.

As for E. von Glasersfeld, he first met constructivist ideas in Dublin during the 1940's, where Edward Sheehy and Gordon Glenavy introduced him to the "real" Berkeley. Then, for nearly two decades, it was the friendship and teaching of Silvio Ceccato and the original group of the Italian Operational School that gave his thinking a radical bent. A third and more recent debt he owes to C. D. Smock, who patiently and delicately guided him to see the constructivism in Piaget and, thus, to look at the problems of knowledge from a developmental point of view.

Introduction

E. von Glasersfeld and C. D. Smock

The purpose of this collection of essays is to suggest an alternative way of looking at knowledge, knowledge acquisition and the process of cognition. The approach is neither easy nor comfortable. It involves the demolition of several concepts which are part of our civilization's "common sense." Many will argue we cannot do without these concepts. But, constructivism does offer a theoretical explanation for the strange and disheartening fact that knowledge, no matter how clear and obvious it may be to the knower, cannot be simply communicated. It does this on the strength of a radical reassessment of the concept of knowledge, a reassessment which, though it is by no means a philosophical novelty, is contrary to the mainstream of philosophical tradition.

The idea that knowledge cannot be taught but must be constructed by the learner is not a new one. A fragment of the pre-Socratic Parmenides in the 5th century B. C. says that "being and thought are one and the same"; and Protagoras coined the often quoted and often misunderstood phrase "Man is the measure of all things." Clearly the suspicion that what we call "knowledge" is in some way the product of the knower had already arisen. For the most part, however, pre-Socratic philosophy was imbued with an enthusiastic interest in the things of Nature and in the type of observation later referred to as "scientific." Suspicions about the universal validity or the "absoluteness" of observational knowledge were not welcome. The Sophists were alone in their view that knowledge is inevitably conditioned by man, "the being that knows and acts" (Capelle, 1935). But even they viewed this as a regrettable limitation. The foundation of the theories of knowledge that have come and gone since was shaped by the Observers of

Nature, with their optimistic common sense attitude. Yet, the Sophists' suspicions about knowledge and the process of knowing were never quite eradicated.

The interest in Nature, slowly but persistently fed by the inquisitive spirits of the Renaissance and formally established as "science" by Galileo, became predominant in the second half of the 17th century. Newton's mechanics, then, was celebrated as the ultimate explanation of the real world and as the purest and most reliable form of knowledge. Perhaps it is no coincidence that, at about the same time, the suspicion surfaced again that knowledge and explanation might have more to do with the knower and the explainer, than with what was being known and explained. George Berkeley (1710) and the little-known Neapolitan philosopher Giambattista Vico (1710) formulated theories of knowledge which, though different in many details, agreed with the Parmenidean fragment that "being and thought are one." Again, it was an unpopular epistemology, and it still is, for the most part, unpopular today. Since then, however, two things have happened which, to us, seem particularly relevant to a discussion of what "knowledge" is, how it is acquired, and how its acquisition might be enhanced.

First, there was Kant's monumental effort to formulate the epistemological problem in a consistently rational way so that all questions that in any sense touched upon it might be answered rationally. We do not intend to assess Kant's contribution to contemporary thought nor are we competent to do so. One fundamental, and simple, point, however, serves as a stepping stone towards many ideas expressed in the essays written for this volume. Man's experience of the world, according to Kant, is necessarily within a framework of space and time; i.e., space and time are inherent in man's way of experiencing. They are characteristics of the human organism in much the same sense in which, say, a lens and a focal plane are essential parts of a camera. If we accept Kant's view, it is clear that to gain objective knowledge of the world, we should have to subtract space and time from our experiences.

And that is not all. Even if we assume that all spatial and temporal aspects could be pruned from an experience, we should then have to be able to become aware of the pruned experience in a way that is not like our usual way of experiencing, in that the experienced item must not be placed into the framework of space and time. Clearly, this would be quite impossible, unless we give up the idea that our way of experiencing necessarily adds a spatio-temporal framework. In other words, if we adopt Kant's view of experience, we may believe that there is an objectively real world, but we cannot possibly tell how it is or what it is like, because what we can represent to ourselves, what we "know" rationally, is the result of our specific way of experiencing.

It should be emphasized that all this concerns "rational" knowledge; that is, knowledge that has some form of invariance, that can be intersubjectively replicated, and about which we can communicate. There may well be other, more "direct" forms of experience that yield results which are not within the spatio-temporal framework; whatever they may be, we should not call them "knowledge" in the rational sense with which we are concerned here.

The second event which has modified the basis of epistemological discussions was the conceptual revolution in physics. This revolution has significantly changed the balance of parties in any debate on the nature of knowledge and it has turned the "constructivist" suspicion into an approach that now seems as tenable - if not more so - than any other.

When Berkeley and Vico developed their ideas at the beginning of the 18th century, they did it not only against the common sense views accepted in everyday life (which, where epistemology is concerned, are probably no different today), but also in direct opposition to the theory of knowledge represented by the work of the great physicists that determined the intellectual climate of their time. In our century, however, it was precisely the work of the great physicists that has undermined the traditional assumptions of epistemology and that, once again, has focused interest on the constructive activity of the "being that knows and acts."

The authors of the essays presented here are all proponents of this shift of focus. Their main interest lies in man, the knower, and in their view "knowing" and "acting" are never detached one from the other.

We have called this school of thought "Radical Constructivism," because it attempts to coordinate "constructivist" ideas from a number of very different sources and, above all, to strip them of the compromises with which they are encrusted. Each of our authors treats constructivism in his own way and from his own particular perspective. To a large extent the essays are concerned with different aspects of cognition and with a variety of cognitive structures. Yet, the authors agree in their radical interpretation of the concept of knowledge. The reader should keep this point in mind, for without constant awareness that "knowledge," in these essays, is defined in a wholly unconventional way, most of what is said will be misunderstood.

"Knowledge," not only in common usage but also in most of the current psychological and philosophical literature, is always tacitly assumed to be knowledge of an existing world. That is to say, what we know is assumed to be an aspect of an independent reality, a reality that exists by itself and in itself. For the traditional philosopher, as well as for the man in the street, knowledge seems inextricably tied to ontology; thus, knowing is a process of finding out what is. From the constructivist point of view, the assumption of a world of being, of ontological reality, makes epistemology a circular enterprise. As Maturana (1970, p. 2) has recently put it: "...the *a priori* assumption that objective knowledge constitutes a description of that which is known...begs the questions *what is to know*? and *how do we know*?"

To give up the belief in the possibility of gaining objective knowledge of an absolute reality is a step most of us are reluctant to take. In Piagetian terms it is perhaps the ultimate, most difficult task of decentration. To move the center of the universe to some unknown place, far away from our little planet, was no doubt an awesome step for those who first made it. To relinquish the idea of an all-powerful father who, if the need arose, would protect us, is still a difficult step for many. The belief in an independent but discoverable world may be the last anthropocentric fiction we have to give up in the pursuit of the intellectual evolution we have embarked on. Everyone would like to think that what he experiences is the "real" world and the only world there is to be discovered. The stress is on "discoverable" - that is the point that is so easily misunderstood. When Berkeley's critics scoffed at his suggestion that what we call "to be" is no more and no less than "to be perceived," they were quite unable to get away from the idea that perception must under all circumstances have an object, a something that is *there* before the activity of perceiving begins. Yet, insofar as Berkeley can or, rather, must be interpreted as a constructivist, that is not at all what he meant. Much earlier he said: "The existence of any thing imaginable is nothing different from imagination or perception" (Berkeley, 1706/1930, p. 97). "Existence," for him, is the *result* of perception; or, as Piaget might say, the result of perceptual construction of invariances. Perception and cognition, thus, become active and creative instead of passive and receptive. The cognitive structures, that result from this perceptual-conceptual activity of knowing, "exist" in their own right and not as the cognizing subject's replica of independently existing things.

It is important to realize that the constructivist model of cognition does not attempt to supply ontological answers. It avoids the traditional concept of being and, therefore, can hardly be compared to solipsism. The constructivist does not believe that there is no world beyond what he perceives and builds up into cognitive structures, but he does believe that this world is not accessible to his rational cognition.

Many scientists and educators may agree that the radical constructivist view is correct "in principle" but not relevant to "practice" much in the same way that Einstein's relativity may be a better, more comprehensive theory, but nevertheless Newtonian mechanics "works" in everyday life for most people. But some argue that a constructivist's epistemology has implications for both the scientist and teacher because it leads directly to the specific proposition that knowledge cannot be taught but only learned (i.e., constructed). Cognitive structures are never passed ready-made from a "teacher to a pupil" (or from "nature" to scientist) because cognitive structures, i.e., knowledge, must under all circumstances, be built up by the learner.

The belief in an absolute reality has been the source of many of the disappointments of educators as well as students. Many things we know are so obvious to the knower that it seems inconceivable that it should be difficult to communicate them to the novice. Mathematics is perhaps the most striking example. Once we have acquired a piece of mathematical knowledge, we are no longer able to see how one could possibly fail to acquire it - it seems so patently obvious. Yet, mathematics educators constantly run into difficulties and their experience is often thoroughly disenchanting. Communication does not seem to work, and so the first reaction is to search for other, better ways of communicating. Rarely, if ever, is the question brought up whether knowledge is, in fact, communicable in the usual sense of that term.

Many of the fads and foibles of educational practices, at all levels, are recurrent simply because of the failure to clearly conceptualize the epistemological bases of instructional theory and practice. The recent return to "discovery" methods, "open" classrooms, and activity-based curricula, accentuates, for example, the need for modification of teacher training programs in the direction of consistency with a constructivist epistemology. It is doubtful whether any technology (materials and methods) can be invented that will prevent teachers trained as "absolute realists" from using these "innovations" in such a way that the intended objectives can not be achieved.

In the essays that constitute this volume, the teacher will find few if any directives that could immediately be applied to his didactic activity. Indeed, if he happens to believe that the methods he has been using are close to optimal (since they are the distillate of innumerable generations of educators) and that their occasional failure is invariably due to the waywardness or fecklessness of his students, this book is not for him. If, on the other hand, he has come to feel, as so many have, that the correlation between the traditional teaching methods and educational success is at best fortuitous, and if, therefore, he has begun to cast about in search of new concepts for a theory of instruction, he may find in these essays some of the raw material he has been looking for.

Von Glasersfeld introduces the reader to the basic principles of radical constructivism in the context of Piagetian epistemology. Drawing on philosophical as well as scientific precedents, he argues for a general shift of paradigm (Kuhn, 1970) with regard to the theory of knowledge and advocates a shift of focus from ontological considerations to the study of knowledge as construct and, hence, to the study of cognitive processes.

The implied conceptual shift from the "organism-as-black-box" to the "environment-as-black-box" is extremely difficult to carry through, even if one is inclined to try. The conventional philosophical ballast of language and the ambiguities of linguistic communication are increased and compounded by the reluctance of our cognitive apparatus to overcome our naive realist beliefs and conceptualizations. As von Glasersfeld clearly demonstrates, terms such as "object" and "environment" are indispensable for communication -- but what meaning should we attach to these terms? Even if clear definitions are given early on, some pages later the reader often reverts to assimilating the terms to his preconceived cognitive structures. Until the constructivist epistemology becomes common sense and natural to us, we shall have to check constantly to determine the conceptual consistency of statements regarding the nature of knowledge and the optimal conditions for the acquisition of knowledge.

Hugh Gash explores similarities in the theories of knowledge propounded by Jean Piaget and John Dewey. That there are certain similarities has been noted by several cognitive developmental psychologists. Gash carries out a more critical comparative analysis. Dewey's epistemology is presented as an account of the dynamic conceptual organization of purposive interactions with experienced sense data. The verb "to know," which in the philosophical tradition denotes the passive reception of "knowledge" and thus a purely metaphorical activity, is here given the meaning of an active process, i.e., a true activity on the part of the knower. This interpretation is all the more plausible since it springs from an historical context characterized by Bridgman's operationalism and Heisenberg's Principle of Uncertainty. One of the results is a distinction between the world as it "is," which we cannot know, and the knowledge of a world which we construct. Gash argues that previous statements of the similarities between the epistemologies of Dewey and Piaget have not made explicit the radical nature of their theories, in particular the peculiar status they confer to the term "environment." There are, of course, also differences between the theories of the two thinkers, such as Piaget's "structuralism" and his distinction between figurative and operative processes. But Gash suggests that a fuller understanding of the basic similarities in these two theories of knowledge may help educators to become aware of the potential a constructivist view of knowledge might have for the educational process.

John Richards examines the nature of "knowledge" in modern mathematics. The mathematician's enterprise, he says, is, after all, a human activity and, as such, it is dependent upon human cognition. If cognitive processes are, indeed, processes of construction rather than of replicating or depicting an a priori existing reality, then the focus of any explanatory effort must shift from what there is or may be to how we arrive at the conceptual constructs we actually have. Richards' discussion, consequently, develops the dichotomy of a "logic of discovery" and a "logic of reconstruction." Though the author does this through a review of the recent divergent trends in the philosophy of mathematics, his findings should be of great interest to teachers of mathematics. On the one hand, it would seem that the logic discovery, once it is explicated, should be wholly compatible with what

Ceccato (see his contribution to this volume) calls the awareness of mental operations. On the other, it follows from Richards' conclusions that the method of mathematical instruction cannot be the imparting of mathematical "truths" but must, instead, be the setting up of circumstances which will induce the learner to achieve in his own mind - i.e., to reconstruct - the conceptual entities and relational functions the mathematicians have "discovered." The logic of reconstruction, thus, becomes crucial to the didactic endeavor and Richards' analysis suggests that to ensure the learner's reconstruction, a good deal more is needed than the mere description of the mathematical constructs he is to acquire.

William Powers, who came to cybernetics as a physicist and electrical engineer, arrived at a constructivist position through purely technical considerations. Originally he was not at all concerned with epistemology or with philosophy in a wider sense. From designing models for the monitoring (and thus prediction) of the motion of stars he proceeded to the design of models for the prediction of organismic behavior. His shift of interest was similar to that of the pre-Socratics. Unlike the natural scientists of the 5th century B.C., however, he quickly became aware of the crucial difference such a shift of interest implied: while the physicist-engineer can afford for a long time to remain a naive realist, because he is exclusively concerned with items that are all wholly within his experiential field, the investigator of living organisms very soon runs into the problem of perception and, if he is consequential, into the problem of how he himself perceives and observes the organisms he wants to investigate.

From the observer's point of view, another organism quite naturally is in an environment which is no less observable than the organism. That is why the observer can close the loop from the organism's "output" to the organism's sensory "input" by a cause-effect link outside the organism, i.e., a link in the "external world." But the moment the observer considers himself and begins to investigate his own organization, he can no longer observe a boundary between organism and environment because there is no way for him to step across his own experiential interface. His "external world" thus can consist of nothing but the collection of invariances he is able to establish between the data he registers as "output" and those which he registers as "input." These invariances are the organism's constructs and, as such, necessarily the result of inductive inference. As Maturana (1970, p. 39) puts it: "A living system, due to its circular organization, is an inductive system and functions always in a predictive manner: what happened once will occur again."

Being an engineer, Powers was perhaps particularly sensitive to the requirement of economy in explanation and predictive modelling. He could not be satisfied with the traditional behaviorist model of organisms which, because it is dependent on an observable environment, cannot be generalized to include the observing organism. His negative feedback model develops in the opposite direction: it starts as a model of the observer and is then generalized by treating any observed organism as an observer in its own right, maybe simpler than the human observer but in any case operating along similar lines. This, indeed, is what educators could glean from Powers' work. If the knowledge a feedback system can acquire is the result of induction, it cannot be passively assimilated in any way by a student, the teacher can only try to create circumstances that will lead the pupil to make the required inferences.

Paul Silverman applies the model of the "feedback loop" to cognitive development and the organism's construction of "invariants." Radical constructivism, for him, is an epistemology that enables the scientist to synthesize his experience of his own purposiveness with an explanatory system for that experience. The strongest argument for constructivism, he maintains, springs from the internal consistency of the explanations it generates. Pursuing the parallelism between Piaget's definition of "knowledge" (i.e., structures that remain constant though their component parts may change) and the idea of converging definitions that derives from Bridgman's operationalism, he argues that both can be understood as a feedback system. Piaget's theory of cognitive development is built upon the concept of equilibrium (homeostasis). Scientific theories - and that is scientific knowledge - live, develop, and die as a function of their internal consistency; i.e., by the relative absence of contradictions. And contradictions, in a very real sense, are disturbances of the logical equilibrium. The model of the feedback loop, thus, is germane to both, because its function is precisely the elimination or neutralization of disturbances with regard to a given parameter or "reference value." Though Silverman does not expound the educational implications of his analysis, these implications are clearly compatible with the basic notions that are made explicit in the essays by Ceccato and Smock.

Silvio Ceccato, one of the European pioneers of cybernetics and Director of the Center for Cybernetics and Language Analysis of the University of Milan since 1949, considers himself a technician of the mind rather than a philosopher. His operational analyses of mental activities are an attempt to supply the constructivist model of "knowledge" and "reality" with a base of well-defined, repeatable, and even mechanizable operations. While other constructivists (e.g., Vico, Bridgman, Dewey, and also Piaget) rarely, if ever, get down to the level of the elementary operations by means of which the organism coordinates his experience to form invariances and, eventually,

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a construct that might be called "reality," Ceccato has from the very beginning pursued the question: How is it made and what does an organism do to make it?

Though his main work is theoretical and makes difficult reading, his contribution to this volume deliberately avoids the more complicated technical issues. His message, here, springs directly from his recent work with school children and embodies in an extraordinarily lucid way the approach to education which modern constructivists are propagating. In simple terms it could be expressed by saying: Since knowledge is the result of construction on the part of the knower, let us try to make children aware of how the mind constructs.

Charles Smock, on the strength of many years' experience with Piagetian theory and research, as well as with educational programs, discusses the role of the teacher and the problems of the educator in general. Though in his conclusions he agrees with Ceccato, he presents, as it were, the other side of the coin. Where Ceccato says, "as far as the teacher is concerned, what is needed is only patience and a little fantasy," Smock is at great pains to show that it is not a simple matter to guide teachers towards the kind of patience and fantasy that create a climate in which the children's construction of knowledge can proceed unhampered. In spite of the fact that his essay was written before the others, it serves well to demonstrate how theoretical considerations of epistemology are, indeed, not merely relevant but quite indispensable if we want to attain a productive methodology in education. His eloquent plea for a constructivist attitude in teachers should go a long way towards convincing any active educator that it would be worth trying a new paradigm of instruction and that the laborious and so often fruitless efforts to impart knowledge might be successfully superseded by attempts to show the student how he can construct for himself the knowledge that is considered the backbone of our civilization.

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Piaget and the Radical Constructivist Epistemology[1]

Ernst von Glasersfeld

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> We do not experience things; things are a construction of ours the function of which is to emphasize the resemblance between aspects of our present immediate experience and aspects of our past experience, something which it proves enormously useful to do.
>
> P. W. Bridgman, 1936

Eight years ago, in his Introduction to Piaget's Six Psychological Studies (1967), David Elkind began with an observation that is both perceptive and ambiguous: "Although Jean Piaget could legitimately lay claim to being a psychologist, logician, biologist, and philosopher, he is perhaps best understood as a genetic epistemologist."

The phrase "he is perhaps best understood" could be taken to mean that Piaget is, in fact, well understood as an epistemologist, and not so well as psychologist, logician, etc. But that would not have been true in 1967, nor, indeed, would it be true today. Instead, we have to interpret the phrase as an exhortation: In order to understand Piaget, one had better consider what he has to say about epistemology. - In the course of his Introduction, Elkind makes it abundantly clear that this is what he intended, and his exhortation is no less pertinent now than it was seven years ago. In much of what has since been written either pro or contra Piaget, there is little evidence that his "Genetic Epistemology" has been understood.

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[1] A preliminary version of this paper was presented at the Third Southeastern Conference of the Society for Research on Child Development, Chapel Hill, N. C., 1974.

Difficulties of Interpretation

There are several reasons for this lack of comprehension. Piaget expresses himself with none of the clarity and precision which, we are often told, are inalienable features of the French language. His translators, therefore, often face what seem insurmountable problems. Also - and this, of course, is my subjective judgment - Piaget himself has for a long time understated the import and the reach of his more radical epistemological ideas. His works are, in fact, full of formulations that give the reader a spurious sense of security. Words such as "perception," "reality," "environment," "object," and "cognition" are frequently used without any indication of the very special epistemological status Piaget has given them in the passages where he elucidates them as terms of his own constructivism. In the last decade, however, he has made the revolutionary aspect of his ideas a good deal more explicit and there is no longer any doubt that much of what has been said in the past about Genetic Epistemology will have to be revised.

Revision may, indeed, be too gentle a word for the kind of reorganization of ideas which, I believe, is indispensable for an understanding of the theory of knowledge which Piaget's constructivist formulations entail. It is not a question of merely adjusting a definition here and there, or of rearranging familiar concepts in a somewhat novel fashion. The change that is required is of a far more drastic nature. It involves the demolition of our everyday conception of reality and, thus, of everything that is explicitly or implicitly based on naive realism; it shakes the very foundations on which 19th century science and most of 20th century psychology has been built, and it is, therefore, not at all unlike the change that was wrought in physics by the joint impact of relativity and quantum mechanics. The fact that Piaget himself now and then reverts to formulations which imply an earlier

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"interactionist" view (Piaget, 1972, p. 17) and which are logically incompatible with the constructivist ideas expressed elsewhere, may make it difficult to guess what Piaget actually believes, but it does not invalidate the logical coherence of radical constructivism as a model of human experience.

Early on, in the first of the "Six Psychological Studies," we read:

> The period that extends from birth to the acquisition of language is marked by an extraordinary development of the mind. ... This early mental development nonetheless determines the entire course of psychological evolutions. In fact, it is no less than a conquest by perception and movement of the entire practical universe that surrounds the small child. At eighteen months or two years this 'sensorimotor assimilation' of the immediate external world effects a miniature Copernican revolution. At the starting point of this development the neonate grasps everything to himself - or, in more precise terms, to his own body - whereas at the termination of this period, i.e., when language and thought begin, he is for all practical purposes but one element or entity among others in a universe that he has gradually constructed himself, and which hereafter he will experience as external to himself. (Piaget, 1956, pp. 8-9)

It would be easy to read this passage and put it aside as containing nothing particulary unusual. True, there is talk of a "conquest by perception and movement," a "Copernican revolution," and of a "universe that he (the child) has gradually constructed" - but we are so often bombarded with pompous nonsense and esoteric metaphors that we have become quite accustomed to the need to separate what matters from the overblown phrases. In this particular case it is also much more comfortable to pick out what happens to fit our normal way of thinking and to consider the rest the kind of emphatic noise that is not at all uncommon in the more old-fashioned European writers.

Taken seriously, a statement to the effect that the child constructs his universe and then experiences it as though it were external to himself, would be rather shocking. We would all like to be hard scientists, and such an "as though" threatens to pull the rug from under our feet. It smells of solipsism - and that is something to which, by and large, we have developed an intellectual allergy; it makes us extremely uncomfortable, to say the least.

This threat is, I believe, the more serious obstacle we have to overcome if we want to understand Piaget's theory of knowledge. It is made all the more difficult, because Piaget himself sees to it that we rarely meet it face to face. His method of exposition, even in his latest writings, is such that it successfully shields the reader from a direct apperception of the radical aspect of Genetic Epistemology. A good example of this can be found in the same "Psychological Study," immediately before the passage quoted above.

> One can say, ... That all needs tend first of all to incorporate things and people into the subject's own activity, i.e., to "assimilate" the external world into the structures that have already been constructed, and secondly to readjust these structures as a function of subtle transformations, i.e., to "accommodate" them to external objects. From this point of view, all mental life, as indeed all organic life, tends progressively to assimilate the surrounding environment. (Piaget, 1967, pp. 7-8)

By saying this a page before the piece I have quoted first, Piaget makes it all but inevitable that the reader will take as a mere metaphor anything that is said later about the organism's construction of reality. Nothing in this first passage raises the slightest suspicion that the "external world," the "external objects," and the "surrounding environment" are, for Piaget, not exactly the same items as, say, for Skinner or for any other author who has never been troubled by epistemological considerations. Thus it is not at all astonishing that the reader will not take it literally when Piaget, one page later, says that the child himself constructs his universe and thereafter experiences it as external to himself. It is asking too much that the reader, coming to this later formulation, go back to the preceding statement, which he has already fitted into his realist views of the world, and, instead, adjust his ideas to it by generating a constructivist epistemology. Even an experienced and basically sympathetic reader is likely to be misled. Here, for instance, is Harry Beilin's explication of "assimilation" and "accommodation":

> The construction of knowledge, more specifically, takes place through the operation of two general processes under the control of an internal self-regulating mechanism (equilibration). The first of these is the assimilation process. It involves the incorporation of environmental data (through physical or mental activity) into existing cognitive structures. The products of new experience are incorporated into mind only to the extent that they are consistent with existing structures. ... The other process under the control of the equilibrating mechanism is accommodation. This represents the subject's response to external stimulation by which existing cognitive structures are effectively utilized for adaptive purposes by becoming integrated with other internal structures or by differentiating as they are applied to new experience. ... In most situations in which the organism functions, both assimilative and accommodative aspects of development take part, although one process dominates, depending upon the demands of reality. (Beilin, 1971, pp. 88-89)

What is said here about assimilation and accommodation is directly derived from Piaget's own formulations - in the quoted passage as well as in many others. Beilin has successfully "assimilated" it into pre-existing cognitive structures. The changes that have resulted from this assimilation are unobtrusive. Instead of "things and people" we now have "environmental data"; instead of "structures that have already been constructed," we have "existing cognitive structures" (which makes them a little less subjective and transient); and then, of course, we now have "stimulation" and "response" - words which can be relied on to appease any conventional psychologist's doubts.

The result is a definition of the two extremely important Piagetian terms which, while making it much easier to accept Piaget, eliminates every trace of his radical constructivism. Assimilation, according to this explication, involves the incorporation of environmental data into existing cognitive structures; accommodation, on the other hand, involves external stimulation and leads to the adaptation of the organism's internal structures to an environmental reality. - Who would suspect that Piaget is an unorthodox thinker? Indeed, this explication rests on the most solid traditional foundations. The cognitive structures which the organism acquires in this fashion are then considered the organism's knowledge of the world and, inevitably, there is the implication that the more the cognitive structures are adapted to the "environmental reality" the better and "truer" will the organism's knowledge be.

The Realist Dilemma

After twenty-five hundred years of philosophical theories of knowledge, all of which attempted to deal in one way or another with that mysterious process of cognition that is supposed to adapt our knowledge to things as they are in an independent outside "reality," there seems to be no reason why we should balk at a phrase such as "incorporating environmental data into cognitive structures." The educational processes to which we have all been subjected may have failed in many ways, but they have been eminently successful in habituating us to the acceptance of absurdities. Thus, no eyebrows are raised at such a phrase, nor does it prod us at once to ask how environmental data (by definition outside us) might get through what we should call the experiential interface so that they can be incorporated into our cognitive structures; or, similarly, how we could possibly accommodate our cognitive structures so that they become more adapted to the "demands of reality," when the structures are, by definition, in us, while the "reality" is supposed to be on the other side of the interface.

Psychologists, with the exception of George Kelly (1963), have preferred to ignore this basic puzzle. That is, I am sure, why Piaget does not particularly like to be considered a psychologist. He has persistently struggled with these questions and, though his exposition is far from exemplary, he does, I believe, imply an answer that makes possible a viable model of cognition. The key to this model is revolutionary, not because it leads to a new theory of how we come to have what has hitherto been called "knowledge," but because it radically changes the very concept of knowledge. For that reason, as Elkind recognized, Piaget must be understood, first of all, as an epistemologist.

Ever since Plato the activity of "knowing" or "cognizing" has been viewed as a kind of copying or replicating (Ceccato, 1949; Piaget, 1961, 1967, 1968). The cognizing subject was thought to acquire or build up inside himself a replica or image-like representation of the outside things, i.e., the "real" object, which he was getting to know. Thus, the outside became a reality to be discovered; and, quite inescapably, it had to contain the things which the subject had already replicated and got to know, as well as those which the subject might get to know.

"When I perceive I must become percipient of something, ... the object, whether it become sweet, bitter, or of any other quality, must have relation to a percipient," says Socrates (Plato, 1949, p. 22) in his rather specious attack on Protagoras; and since then we have lived with the problem of how to specify or describe this very peculiar relation between the, as yet, unknown but existing object on the one hand, and, on the other, the subject that has to replicate this object in his head in order to know it. The senses, quite naturally, were given the task of mediating between the "real" object and the subject's representation of it. - But this immediately raised the question as to how the subject could ever be sure of having an "adequate" or "true" representation, if the only way he can get at the object is through the mediation of his senses. Indeed, there is no rational way around the logical difficulty that the senses cannot possibly test the veracity of their own products. The careful, Platonic subject, hence, speaks of the "illusory appearance" of things and, to gain some kind of certainty, falls back on "innate ideas." The more self-confident positivist, taking a cruder path, cavalierly ignores the problem and proclaims that the world simply has to be the way we see it.

The Construct of the "Object"

Piaget approached the problem of cognition from an altogether different angle. Before attempting to deal with highly complex conceptual items such as "reality" or "environment," he asked rather modestly, how does a child come to have the concept of an "object" that has some kind of permanence in his stream of experience. Piaget's answer is common knowledge today and can be found, in one form or another, in any of the more recent introductory textbooks of psychology. Whichever way it is formulated, it is generally understood that "object permanence" is the result of the subject's coordination of experiential data from more than one source. As a rule, however, the two epistemologically most important aspects of this subjective coordination are not made very explicit.

The first of these was stated in an approximate way already by Ernst Mach before the turn of the century (Mach, 1886) and summarized in one of his last papers.

> I can see an object if I look at it, I can feel it if I touch it. I can see it without feeling it, and vice versa. As a rule, however, visibility and tangibility are linked. Although the emergence of the elements of this complex takes place only on certain conditions, these are so familiar to us that we hardly notice them. We regard an object as being always present, whether or not it is sensible at the time. We are accustomed to regarding the object as existing unconditionally, although *there is no such thing as unconditional existence*. (Mach's italics) ... to extrapolate this experience beyond the proper limits of experience, and to assume the existence of a "thing-in-itself," has no intelligible meaning. ... We have become accustomed to regarding an object as existing permanently. (Mach, 1970, p. 30)

Stressing, as he does, the subject's active part in making objects "exist permanently," Mach must be considered a forerunner of modern operationalism as it was launched by Bridgman in 1927. Bridgman, in fact, explains "object permanence" in a way that is identical with Piaget's analysis:

> When we say that we see a thing out there in space we are exploiting correlations built, by experience and repetition, into the structure and functioning of our brains. (Bridgman, 1961, p. 46)

Both Mach and Bridgman were concerned mainly with the "existence" of objects, and they came to the conclusion that this "existence" is the result of our own constructive coordination of experiential data and our subsequent projection of these coordinations into an "outside" world. Neither Mach nor Bridgman considered the developmental aspect of the subject's coordinatory activities. This is what Piaget has done, and his empirical findings concerning the genesis of object permanence in infants are a splendid example of the experimental confirmation of an idea postulated on purely theoretical grounds.

Besides, Piaget has made the process of coordination a good deal more explicit. When Mach said, "I can see an object if I look at it, I can feel it if I touch it," he gave no indication that this "seeing" and "feeling," too, must consist in coordinatory operations on the part of an *active* subject.

That is to say, shape, form, or, generally speaking, pattern must be considered no less the result of a subject's coordinatory activity than the concept of a permanent object.[2] As far as I know, Ceccato, the founder of the Italian Operationist School, was the first to propose this "radical" constructivism, and he did it on purely logico-theoretical and not, as Piaget, on developmental grounds. We call this school of constructivism "radical" because it holds that the knower's perceptual (and conceptual) activity is not merely one of selecting or transforming cognitive structures by means of some form of interaction with "existing" structures, but rather a constitutive activity which, alone, is responsible for every type or kind of structure an organism comes to "know." And this brings us to the second epistemologically important point in the analysis of object permanence.

The Building-Blocks of Construction

Following the philosophers' traditional use of the expression "sense data," psychologists rarely discriminate the two possible meanings of the term. They tacitly assume an epistemological position that is nothing but a reductionist brand of realism and, therefore, incompatible with any radical constructivist theory of knowledge. If by "sense datum" we mean a characteristic (primary, minimal, elementary, or whatever) of an independently existing object, i.e., an item that possesses that characteristic independently of the subject's sensation, we are still faced with the unanswerable question whether the datum, as sensed by the subject, does or does not correspond to a "real" characteristic of the object. Philosophers, as a rule, perform some sleight of hand to get out of this dilemma; psychologists merely look the other way. - If, however, we take the second, more prosaic meaning of "sense datum," which can be defined as elementary "perturbation" or "signal" on the knower's side of the experiential interface or, simply, as "elementary particle of experience," we can get rid of the realist implications and of the problem of the impossible comparison between an "inside" and an "outside" datum.

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[2] This, of course, does not preclude that certain patterns of visual, tactual, or other sensory data are genetically determined; but if they are "wired in," this does not make them more ontologically real - it merely means that the coordinatory process was carried out phylogenetically by mutation and facilitation of survival rather than ontogenetically by the individual perceiving subject.

Piaget has not been very explicit about this. In the Conclusion of his work on perception (Piaget, 1961), for instance, he frequently talks about the "object" as though he had Kant's noumenon in mind, i.e., an absolute entity of independent "reality," a thing-in-itself which, though perceived only approximately by our senses, must nevertheless have a structural correspondence to the phenomenon we experience sensorially. It all sounds as though, in spite of his genetic analysis of the child's construction of object permanence, he still felt the need to see the constructs of experience as replicas, at least in a structural sense, of really "existing" objects.[3] However, in the light of what Piaget has said about the child's constructive activities during the sensorimotor period ("a universe ... which hereafter he will experience as external to himself") and many of his other statements on the "externalization" of constructs (Piaget, 1956, 1968, 1970), one can hardly doubt that he would agree with Hebb's formulation: "At a certain level of physiological analysis there is no reality but the firing of single neurons" (Hebb, 1958, p. 461).

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[3] This, indeed, is the reason why Piaget is often described as an "interactionist" and why his reader at times gets the impression that the Genevan constructivism does not really go any further than the relatively conservative epistemological models of "active" cognition proposed, for instance, by Cassirer (Die Sprache und der Aufbau der Gegenstandswelt, Berichte des 12. Kongresses der Deutschen Gesellschaft für Psychologie, 1932, pp. 134-145), by Reichenbach (Experience and Prediction, U. of Chicago Press, 1938), Brunswik (The Conceptual Framework of Psychology, U. of Chicago Press, 1952), and others who still cling to some idea of correspondence between cognitive structures and independently "real" structures.

From the radical constructivist point of view, "firings" are, of course, a metaphorical expression for the minimal units of experience; and "neurons," in which they are said to originate, and the "nervous system" that is said to process them, are constructs which, though placed inside the experiencing organism's "body," are no less an externalization or projection beyond the knower's experiential interface than is any "object" in the further reaches of the externalized world. Some such externalization, indeed, seems to be the prerequisite of any rational construction - and presumably also of any form of self-awareness that enables an organism to view his experience rather than merely to experience it.[4] The crucial difference between the realist and the constructivist, thus, is not that the one projects his cognitive structures beyond the experiential interface, while the other does not; the difference is that the realist believes his constructs to be a replica or reflection of independently existing structures, while the constructivist remains aware of the experiencer's role as originator of all structures, i.e., for the constructivist there are no structures other than those which the knower constitutes by his very own activity of coordination of experiential particles.

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[4] Anatol Rapoport (Synthese, $\underline{\underline{8}}$, 1949/51, p. 185) said: "Knowledge consists of an ordered sequence of neuromotor events." - Since knowledge implies a knower, "neuromotor events" necessarily have to be taken as an externalization of elementary experiential particles, or combinations of experiential particles, and not as items existing on the other side of the knower's experiential interface.

Instead of the "firing of neurons," Piaget occasionally speaks of "aliments" or of the "given."[5] These terms, once more, shroud in mystery rather than clarify the basic epistemological issue. If we want to avoid the unanswerable questions to which any version of "replica" theory of knowledge necessarily leads (see above), Hebb's formulation is a much safer one. The basic unit of experience, the "elementary particle" of cognition, in the constructivist interpretation, is itself a construct - if only for the reason that, in direct experience, we are never aware of particles, let alone of the "firing of neurons." But just as Bridgman found "thing" to be an enormously useful construct (cf. quote on p. 1), so do we today, after several decades of neurological conceptualization, find the firing or signal of a neuron an analytically powerful construct. The important point is that such a signal or firing can be taken as a datum in its own right that need not be considered the effect of some independent and intangible cause. The constructivist, who remains aware of the fact that this "elementary particle" is his construct which he imposes upon the flow of experience, may externalize it as the signal originating in a neuron and he may then consider all cognitive structures, no matter at what level of complexity, the results of the knower's active coordination of these signals - and he may thus provide a consistent and non-contradictory analysis of knowledge that does not pretend to reflect in any sense the ontological "reality" of an independent world. Ceccato, who in his early model of mental operations (1961) posited binary flip-flop devices as "differentiators," called the firing signals simply "differentiata." In his later model (1966) he derived not only shapes and objects but also abstract conceptual categories from regularities in the co-occurrence and the hierarchical structuring of particles of attention.[6]

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[5] e.g., "... the necessary and continual internal coordinations that make possible the integration of external cognitive aliments ..." Biologie et connaissance (Gallimard, Paris, 1967), p. 34. (My translation).

From the radical constructivist point of view, then, both the raw material (i.e., the firings or "sense data") and the cognitive structures which become the organism's reality (i.e., the invariant patterns of coordination) are from the very outset "inside" the cognizing system. This might seem to entail an inexorable solipsism - but this impression, I believe, arises solely from the ontological pretensions in which traditional philosophy has steeped us. The constructivist schools, Piaget's as well as the Italian one, are well aware of the fact that no organism is free to construct any reality he might wish to construct and that, instead, there are certain constraints with regard to all construction; but these constraints are not specifiable in terms of ontological characteristics of independently existing "real" structures to which we have no access. Any specification or description of the constraints, therefore, must be formulated in terms of the availability of single, as yet uncoordinated signals (i.e., particles of experience) and of the regularities or interdependence of these signals which the knowing organism, as a result of his own cognizing activity, singles out from his initially undifferentiated continuous stream of experience.[7]

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[6] Ceccato's model of the basic mental processes could, I believe, be considered a cybernetical implementation of the approach to the structure of knowledge worked out by G. Spencer Brown in his Laws of Form (Allen and Unwin, London, 1969); in both systems all structures are hierarchical arrangements of binary "differentiata."

[7] It is the failure to appreciate this aspect of constructivism and the ancestral belief that science must be able to discover the world as it is which leads, for instance, George G. Simpson to refer to "Bridgman's despairing conclusion that 'the very concept of existence becomes meaningless'." (This View of Life, Harcourt, Brace and World, New York, 1963, p. 96). It is apparently extremely difficult for traditional thinkers to separate the concept of "knowledge" from the fiction of "absolute existence" and to realize that this epistemologically necessary separation does not imply the demise of "science."

The Environment as a Black Box

The difference between the constructivist's awareness that all coordination - and thus all structure - is the result of his own activities and, on the other hand, the traditional and common sense view that the cognizing subject in some way discovers structures that belong to an independently existing reality - this difference may seem a subtle one, but it is crucial from the epistemological point of view.

Perhaps the most enlightening demonstration of this point was supplied by the pioneering work of Lettvin, Maturana, McCulloch, and Pitts (1959). On the basis of anatomical considerations and micro-electrode recordings from single fibres of the optic nerve, they established that the frog's visual system has four types of highly specialized "detector" networks, one for sustained light-dark constrast; a second for small dark convex shapes; a third for a moving edge; and a fourth for sudden dimming of illumination. The speed with which signals are conveyed to the brain is different for each of the "detectors," and there are certain conditions under which a discharge from one will cancel the discharge from another. The system as a whole makes the frog an efficient fly-catcher, because it is tuned for small dark "objects" that move in an abrupt fly-like way. In the frog's natural habitat, as we, who observe the frog, see it, every item that possesses the characteristics necessary to trigger the frog's detectors in the proper sequence is a fly or bug or other morsel of food for the frog. But if the frog is presented with a black bead, an air-gun pellet, or any other small dark moving item, it will snap it up as though it were a fly. In fact, to the normal frog's visual apparatus, anything that triggers the detectors in the right way, is a "fly."

What are the epistemological implications of this frog story? The simplest way of putting it is perhaps this: Whatever is perceived is basically composed of signals within our sphere of experience. We are, of course, free to consider these original signals the effect of some outside causes. But since there is no way of approaching or "observing" these hypothetical causes, except through their effects, we are in the same relation to that "outside" in which the first cyberneticists found themselves with regard to living organisms - that is to say, we are facing a "black box." We may observe and record the "output" from the black box (in this case the "sense data," the signals on our side of the interface), and we may observe and record the "input" to the black box (in this case "proprioceptive data" and "feedback signals," again on our side of the interface); both are neuronal signals - but once we have imposed a differentiation of "input" and "output," we can establish recurrent coordinations and more or less reliable dependencies between the two. Having done this, we can construct an "external world" and our "selves" on the basis of input-output relations.

The history of science and, especially, of technology shows how very far we can get by exploiting such input-output relations and the inductive inferences by means of which we predict future outputs from the black box world on the basis of regularities and invariances in the recorded past. It is a black box with which we can deal remarkably well. As far as its structure, its ontological character, is concerned, it is nonetheless black, and there is no hope of a rational way of dispelling its blackness. We can no more get at the objective character of its "ontological reality" than the frog can get at the objective character of the items to which his visual system reacts. As far as the frog could "know" - if his brain allowed for the complex processing involved in the kind of self-monitoring we call conscious cognition - any constellation of conditions that happens to trigger the detectors of his "fly"-spotting system, be it insect, pellet, or accidental co-occurrence, would be a "fly"; his representation, his concept of "fly" could be defined only in terms of the neuronal signals that concur in the experience and never in terms of the inaccessible hypothetical outside "causes" of these signals.

It should be clear that, in principle, our position cannot be different from the position of the frog. To say that it is, and to argue that we can discover aspects of an "objective reality," because we are able to experiment and to modify our "environment," is merely to extend the realist illusion. What we experiment with - no matter how elaborate the apparatus and the conversion of different sensory modes - is, in the last analysis, never anything but the interrelation of our signals which we have come to consider input to, and output from, the black box of the "universe"; and what we modify or control by our activities are always, as William Powers (1973) has formulated it, our own perceptions, i.e., the signals we call sense data, and the ways in which we coordinate them. We may and do, of course, project structures that result from this coordinating activity into an "outside," but even the most spectacular successes we achieve in predicting and controlling our experience give us no logical ground whatsoever for the assumption that our constructs correspond to or reflect structures that "exist" prior to our coordinatory activity. Even if we posit causes for the sense data (i.e., the particles into which we can break up our experience), this does in no way entail that these causes exist in the spatio-temporal or other relational structures into which we have coordinated them. The fact that we can coordinate our own sense data into recurrent structures can never prove that these structures are ontologically real - it only proves that the individual data occur frequently enough in our experience for us to establish "invariant" co-occurrences. There may, indeed, be countless ways of operating and arriving at coherent structures that are no less recurrently imposable on our stream of experience than the ones we have come to construct. To disregard this, and to attribute ontological status to our constructs is precisely what Kant, in one of his mellower moments chidingly called "dreaming idealism" (1950); when scientists go in for this kind of dreaming it might deserve a harsher word.

The integrated hierarchical feedback theory that Powers has developed bridges the conceptual gap which, hitherto, made it almost impossible for a traditional psychologist to accommodate to the constructivism of the Piagetian or of the Italian school. In the first place, Powers (1973) begins by pointing out the insufficiencies and unwarranted assumptions in Brunswik's (1952) epistemological model, a model that is more familiar to American psychologists than the philosophical precursors of either Piaget or Ceccato. Secondly, Powers' demonstration that a system's behavior can be better understood as modifying the system's perceptual "input" rather than as the result of it, is a good deal simpler and more economical than any philosophical argument. It makes the epistemological point without talking about epistemology. Thirdly, the use of well-known cybernetical terminology makes Powers' work a great deal more accessible than either Piaget's or Ceccato's. When Piaget says:

> In reality, the element to which we must constantly turn in the analysis of mental life is "behavior" itself, conceived, as we have tried to point out briefly in our introduction, as a re-establishment or strengthening of equilibrium. (Piaget, 1967, p. 15)

it is not at all easy to glean from this the underlying idea that behavior aims, not at modifying, some unknown and unknowable thing or event in an hypothetical outside world, but aims at diminishing the difference between the present coordination of sensory signals and a previous construct. This idea is nevertheless implicit in Piaget's concept of "equilibrium," which as he reiterates often enough, is the result of both assimilation and accommodation. But the way he formulates this frequently suggests to the realist reader that what is being balanced, by means of assimilation and accommodation, are the organism's cognitive structures on the one side of the scales, and the "demands" of a real environment on the other. A careful reading does, I believe, show that this is not what he intends.

Already in his volume on perception (1961) Piaget says quite clearly:

> Knowing consists in constructing or reconstructing the object of knowledge in such a way as to grasp the mechanism of this construction; which is the same as saying (if one prefers to use the terms which positivism has persistently but ineffectually proscribed) that to know is to produce in thought (i.e., in the thinking mode), and the production must be such that it reconstitutes the way in which the phenomena are produced. (Piaget, 1961, p. 441-442) (My translation).

There is, of course, a strong echo in this, of Giambattista Vico (1711), who first maintained that man can know only what man himself has made (i.e., produced, constructed), and who, therefore, is the acknowledged spiritual father of the Italian Operationist School. - In recent texts on epistemology, Piaget's constructivism is even more explicit:

> By contrast, for the genetic epistemologist, knowledge results from continuous construction, since in each act of understanding, some degree of invention is involved; in development, the passage from one stage to the next is always characterized by the formation of new structures which did not exist before, either in the external world or in the subject's mind. (Piaget, 1970, p. 77)

> ... experience deals with the connection between characteristics introduced by action in the object (and not on its previous characteristics). In this sense, knowledge is abstracted from action as such and not from the physical characteristics of the object. (Piaget, 1972, p. 31)

Knowledge and Equilibration

Given the definition of "knowledge" that emerges from these passages, we may now ask how Piaget relates the concepts of "assimilation" and "accommodation" to the cognizing organism's continuous activity of construction and equilibration of constructs. The following passage, though it does not contain the actual terms "assimilation" and "accommodation," provides one of the most comprehensive and comprehensible answers Piaget has so far given.

> Rhythms, regulations, and operations, these are the three essential procedures of the self-regulation and self-conservation of structures; anyone is, of course, free to see in this the "real" composition of structures, or to invert the order by considering the operative mechanisms the source of origin, in an atemporal and quasi-Platonic form, and by deriving everything from these mechanisms. In any case, however, it will be necessary, at least with regard to the building up of new structures, to distinguish two levels of regulation. On the one level the regulation remains internal to the already formed or nearly completed structure and, thus, constitutes its self-regulation, leading to a state of equilibrium when this self-regulation is achieved. On the other level, the regulation plays a part in the building up of new structures, by incorporating one or more previously built-up structures and integrating them as sub-structures into larger ones. (Piaget, 1968, p. 16) (My translation).

In somewhat simpler terms, equilibration, in the cognitive realm, involves the adjustment of, for instance, percepts to conceptual structures which the perceiver already has assembled; and this adjustment of the new to the old is called "assimilation." But cognitive equilibration also involves the adjustment of concepts to percepts, and this second type of adjustment, which can take the form of creating a novel structure or of combining several already assembled structures to form a larger conceptual unit, is called "accommodation."[8]

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[8] As C. D. Smock has recently put it: "Equilibration ... is a response to internal conflict ... a matter of achieving accord of thought with itself ..." (Constructivism and principles for instruction, in this volume).

In Piaget's system - and this is the important point that is often overlooked - the "percepts" which I have contrasted with "concepts" in order to explain the two types of cognitive adjustment are not at all what they are for conventional philosophers and psychologists, for they, too, are the result of preceding rounds of assimilation and accommodation. This is the case, not only when the adjustments concern perceptual structures (i.e., coordinations of sensory signals) but it is the case in every act of cognitive adjustment. In other words, assimilation and accommodation are operative on every level of cognitive activity; what differentiates the two is the relative novelty of the constructs to which they give rise.

At the very beginning of an infant's cognitive development, his coordinations of neuronal firings are, in Piaget's view, already subject to assimilation and accommodation. Assimilation, because the actually occurring sensory signals are continually selected and coordinated to fit the genetically determined or "wired-in" structures of perceptual and motor activity. Insofar as this selection and coordination varies from instance to instance to allow for the recurrent application of the fixed structures, assimilation is at work; insofar as the fixed structures are modified, combined, or supplemented in order to fit novel instances of selection and coordination, accommodation is at work. Repetition of any structure results in its perpetuation, in the sense that it becomes a relative fixture that can be used again, once it has been assembled. When this use or application is achieved by an adjustment to the actually occurring sense signals or their coordination, it is a case of assimilation; whereas if it is achieved by an adjustment of the assembled structure, it is a case of accommodation.

Conclusion

Though Piaget's writings make it difficult to establish his basic epistemological position unequivocally - be it because his ideas have continued to evolve in the course of a long life's work, be it because he prefers to coax his readers rather than force them into understanding - he has time and again made statements that go beyond a mere suggestion of a radical constructivist basis; they can be integrated into a coherent theory of knowledge _only_ if we interpret them from a radical constructivist point of view.

The radical constructivist's interpretation of Piaget's Genetic Epistemology, then, consists in this: The organism's representation of his environment, his knowledge of the world, is under all circumstances the result of his own cognitive activity. The raw material of his construction is "sense data," but by this the constructivist intends "particles of experience"; that is to say, items which do not entail any specific "interaction" or causation on the part of an already structured "reality" that lies beyond the organism's experiential interface. As a cognitive construct, this "interface" is a corollary of the organism's externalization of his constructs, an operation manifestly inherent in every act of self-consciousness or experiential awareness. Though externalization is a necessary condition for what we call "reality," this reality is wholly our construct and can in no sense be considered to reflect or represent what philosophers would call an "objective" reality; for no organism can have cognitive access to structures that are not of his own making.

Cognitive equilibration must be viewed as a kind of ideal state that is never achieved. The organism works towards it by assimilating the signals he is actually coordinating at a given moment (or stage) to the structures he has formed in the past; and he also works towards it by accommodating the already formed structures, whenever the signals with which he is operating cannot be fitted into one of the available structures as they are.

What saves this epistemological model from absolute solipsism is the constructivist concept of "adaptation."[9] Once again, however, we must be careful not to interpret this term in the way current among realist biologists and psychologists. What the organism adapts to, and what ultimately determines the pragmatic viability of his constructs, are certain regularities in the input-output relations the organism registers, with respect to the black box which he experiences as his "environment" or "world." Since the constructivist holds that all coordination and, therefore, all structure is of the organism's own making, he remains constantly aware of the fact that, though he may project his constructs beyond his experiential interface, he must not and cannot consider them ready-made structures of an ontologically prior world. The structures he calls "things," "events," "states," and "processes" are the result of the particular way in which he himself has coordinated his "particles of experience"; and the fact that, at a certain level of elaboration, he can assimilate a good many "perceptual events" without further accommodation of his cognitive structures, tells him nothing about the onotological reality of these "well-adapted" structures. At best - and if he chooses to externalize structures such as "neurons," "firings," and a "central nervous system" - he may say that the universe is such that it will supply whatever conditions trigger his "receptors" frequently enough for him recurrently to coordinate their firings, and to keep the thus coordinated structures relatively permanent. He may then call this relative permanence of certain structures "reality" - especially once he has managed to elaborate his cognitive construction to incorporate "others" and "communication."

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[9] For a somewhat different argumentation with regard to solipsism, see Heinz von Foerster, On constructing a Reality, in Environmental Design Research, W.S.E. Prizer (Eds.), Dowden, Hutchinson, and Ross. Stroudsburg, 1973.

As we know well enough from our own experience, at that level of elaboration, the "permanence" or "universality" of our cognitive constructs tends to become precarious. But even if it did not, even if we could achieve perfect intersubjective agreement of structures, it still would not get us across the border of the black box, because all it would tell us with certainty is that we, collectively, have found one viable construction.[10] Such a construction becomes no more "real," in the ontological sense, if we share it - it would still be based solely on signals on our side of the construct we have called "experiential interface," and on the particular way in which we have categorized, processed, and coordinated these signals as input to, or output from, the construct we have called "universe."

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[10] This aspect of constructivism has been extensively treated by Humberto Maturana in his The Biology of Cognition, Report No. 90, Biological Computer Laboratory, Department of Electrical Engineering, U. of Illinois, Urbana, 1970.

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The Constructivist Epistemology in John Dewey, Jean Piaget, and Cognitive Developmental Psychology

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Introduction

There are affinities in the descriptions of the acquisition and nature of knowledge which have been presented by Dewey and Piaget. Kohlberg (1969, 1971) and Kohlberg and Mayer (1972), who are developmental psychologists, have noted these similarities. However, their statements of this association neither explore the extent of the similarities or the nature of the differences between the theories of Dewey and Piaget; nor do they make clear the radical nature of the constructivist epistemology on which they rest. Since considerable attention is being given to cognitive developmental models at present, it is important to try to clarify these issues. This will be attempted by means of an analysis of the constructivist position in Dewey's epistemology with emphases on the role of action and operations in the process of knowing. The similarities between the theories of Dewey and Piaget can be described in terms of their pragmatist analyses of the role of activity in knowing, and their account of the process of knowing. The differences between the two theories will then be discussed since they have been under-emphasized in cognitive developmental accounts of the similarities between these theories. At the end of the paper, a number of interpretive problems will be discussed.

John Dewey and the Constructivist Epistemology

Dewey (1929) argued that an analysis of the relation between ideas and action at the time of the Copernican revolution shows that a radical change had occurred in the implicit assumptions about the origins and nature of knowledge. This change pointed to the important role of action in the process of knowledge acquisition. The history of epistemology is a long one, and clearly beyond the scope of this paper. However, it will be helpful to describe the accounts of the origins of knowledge which were prior to the synthesis presented in Dewey's analysis. Did knowledge come from sensation, or from thought, or from some combination of the two? Thus, one view held that knowledge derived from sense data which was recorded in a passive mind. Reality was outside, and we knew reality because our senses received data from it. An alternative view maintained that the world was a chaotic reflection of the pure forms of thought, which alone guaranteed truth. In this case, ideas were the structures of our picture of reality. One synthesis of these two positions was available in Kant's philosophy which postulated that knowledge was possible because of the *a priori* conceptual organization of sensations. The constructivist account of knowledge and knowing differs from each of these positions in its emphasis on the role of the activity of the knower in his organization of and integration of his sensations and ideas. A second difference is the denial that the validity of knowledge can be verified by checking with "reality"; this is entailed because "reality" is itself a construction.

The following quote can serve as an introductory statement to the constructivist epistemology:

> The scientific revolution came about when the material of direct and uncontrolled sense experience was taken as problematic; as supplying material to be transformed by reflexive operations in known objects. (Dewey 1929, p. 258).

The implication here is that what is known is the result of the organization of interactions between the individual's ideas and his experience of his surroundings. Further, while Dewey was clearly referring to sense experience resulting from interactions with "material objects" in the quote just given, it is clear that this model of knowing was not to be restricted to such objects, but rather was intended as a general account of the process of knowledge acquisition. Another reason that this quote is useful in introducing the constructivist epistemology is the identification of the process of knowing with the process of science. In fact, Dewey (1929) described his epistemology as a theory of experimental knowing.

Dewey's account of the relation between activities and ideas cuts across the separate status given these terms in both classical epistemology and ordinary language usage. As Dewey pointed out, the common usage of the words "ideas" and "activities" grew out of an epistemology which developed prior to the scientific revolution. Thus, the way in which we separate ideas and action in our everyday language affects our understanding of their interrelatedness. One way of understanding the problem is through an analysis of the early development of the child's thought. Ceccato (1961), who is a cyberneticist, has argued that our understanding of what it is to know was distorted because the activities involved in observation are acquired early in infancy, so it is difficult for adults to observe these activities. Studies on the development of cognition and perception were needed to clarify the dynamic relation between action and knowledge. Another argument presented by Ceccato was that philosophy first began to ask questions about the nature of observation during a period when the tradition of speculative philosophy was firmly established. Under these conditions observation was studied as the observation of objects, which were naturally assumed to exist prior to their observation. Thus, it was not possible to question the independent existence of the reality which is known.

Dewey's analysis of the relation between ideas and action grew out of Peirce's writings on the clarity of ideas. "It appears, then, that the rule for attaining the third grade of clearness of apprehension is as follows: Consider what effects, which might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object" Peirce (1881). Also, while considering the continual growth of knowledge and the emergence of new concepts, Dewey described ideas as tentative hypotheses. These hypotheses were anticipatory plans whose value was discovered in the consequences of testing these plans.

There are two types of operations involved in the actual testing out of ideas, physical operations and symbolic operations. Physical operations are required in order to ascertain the nature of the data received. Thus, activities such as looking, touching, and listening result in sensory consequences. Conceptual searchings through past experience yield ideas which can be used to assess the sensory data. This previous knowledge will also suggest new ideas and new experiments until the additional evidence supports one idea and the problem is solved (Dewey, 1929). The dynamic manner in which data and ideas interact is a central feature of the constructivist epistemology and one which distinguishes it from Kant's epistemology. The following quotes are useful in understanding Dewey's position:

> Original objects of experience are produced by the natural interactions of organism and environment, ... The distinction between sense data and interpretive ideas is deliberately instituted by the process of inquiry, for the sake of carrying it

> forward to an adequately tested conclusion, ... Hence the material selected to serve as data and as regulative principles constantly check one another; any advance in one brings about a corresponding improvement in the other. The two are constantly working together to effect a rearrangement of a new object having the properties which make it understood or known. (Dewey, 1929, p. 173)

Dewey's use of the term operations refers to Bridgman's operationism. Bridgman was a physicist who was writing about the epistemological consequences of post-Heisenberg physics, and he made explicit the pragmatists' account of the relation between activity and concepts. Operations were defined in the following way:

> To find the length of an object, we have to perform certain physical operations. The concept of length is therefore fixed when the operations by which length is measured are fixed; that is, the concept of length involves as much as and nothing more than the set of operations by which length is determined. In general, we mean by any concept nothing more than a set of operations; the *concept is synonymous with the corresponding set of operations*. (Bridgman, 1927, p. 5) (Italics in original)

It is necessary to emphasize that Bridgman's intention was to try to clarify the nature of the relation between activities, words, and experiences. This aspect of Bridgman's operationism was emphasized by Dewey, but has been underemphasized by psychologists who have found the operational definition a useful tool ensuring methodological objectivity. It is important to clarify this latter point because otherwise the use of the word operation will consistently suggest the wrong meaning. For an example of this standard transformation of Bridgman, consider Stotland and Canon:

> The thinking behind such an approach maintains that we mean by such a concept nothing more than a set of operations by which the concept is measured or manipulated in some fashion (Bridgman, 1928). Thus, the concept of intelligence would be defined as whatever it is that standard tests of I.Q. test or measure. (1972, p.2)

In this quote Stotland and Canon have used the operational definition to standardize method. This usage of the term operation is useful, though it is a usage which is not cognizant of the other technical usage of this term as it is found in Dewey's as well as in Bridgman's writings. Generally, psychologists have failed to understand the manner in which operations relate to concepts; in strong contrast, for Dewey concepts were definitions of the consequences of operations.

Bridgman's concern with specifying operations grew out of his dissatisfaction with the manner in which words describe experiences. He felt that language descriptions make experiences appear static, in a manner which is not true to the experience. It is important to remember that our experience is not composed of static items and that it is for convenience that we talk as thought this were so. This convenience results from the correspondence between aspects of present experience and aspects of past experience. However, this convenience may present problems since one is dealing with a construction which doesn't reproduce experience but only certain features of it. The only recourse which constructivists have is to test statements which purport to be knowledge-statements by translating them into testable operations, because meanings are determined by operations.

When concepts are defined in terms of operations, their adequacy can be assessed with reference to the consequences of these operations. When there is a problem in the data one tries to account for it, and whether it is a problem in psychology or in driving one's car, the process of thought is the same. For example, in reflection one forms an idea about the problem and its solution, and then translates this idea into testable operations. The consequences of these operations either confirm or fail to confirm the idea on which they were based. In the case of well known operations, such as driving a car, the reflection will be quite automatic. The notion of testing ideas in terms of their consequences, as opposed to testing them with reference to an antecedent reality, was not possible without an understanding of the relation between purposive activity and knowing. Dewey (1929) maintained that this understanding was absent from epistemologies prior to that of Peirce (1881), whether they were realist or idealist, and irrespective of whether they assumed man to be a passive recipient of environmental information or an active synthesizer.

The ideas which we know are no more than means by which we organize our experience of our environments. It is often difficult to remember that it is logically incorrect to assume that these ideas reflect an objective reality. For this reason Dewey warns against 'reifying' ideas, and exhorts thinkers to try to remember that sense data are taken from our experience of the environment, noting how different the history of epistemology would have been if this had been noticed earlier.[1] It is the directed or purposive nature of our knowing process which limits our descriptions of phenomena; since the ideas are constructed within the constraints of a particular problem, or particular problems, it is always possible that a change in the nature of the problem will entail a corresponding change in the ideas used to organize our experience of the phenomena. Another limitation to our ideas stems from the changing nature of our experience of the environment. This follows from what has been said about the nature of uncontrolled experience, and about the importance of controlling our experience in order to organize it into something known. So ideas may suddenly be found inadequate because of some aspect of our experience of which we are initially unaware. When our ideas are found to be inadequate, we must construct new ones by formulating hypotheses about the inadequacies and testing them operationally.

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[1] It should be noted that Berkeley, the Irish 'British Empiricist,' recognized this (e.g., Of the Principles of Human Knowledge, section 38).

In the constructivist epistemology, knowledge of reality is recognized as being our organization of experience of reality. In this context the problem of the validity of concepts, which traditionally was one of discovering the relation between knowledge and reality, is replaced by the problem of discovering how to test our ideas and intuitions. Thus, within the constructivist epistemology ontological questions are unanswerable and all knowledge is considered tentative and not final.

The view that knowledge is not to be assessed by a comparison with an antecedent reality, but by the consequences of planned operations is the central theme of The Quest for Certainty. The irrelevance of ontological questions is apparent in Dewey's (1929) discussion of Heisenberg's Principle of Uncertainty. From this Principle it followed that either we can decide to control the velocity of particles and incur inaccuracies with respect to mass, or we can control the mass of particles and incur inaccuracies with respect to velocity. In each case we have to recognize that our concepts slightly distort the phenomena described. This is a very serious problem if we have assumed that our concepts reflect reality in an objective sense. And it is the common sense view, and the view of most psychologists, that our concepts do reflect reality in this objective sense.

Dewey described the epistemological consequences of Heisenberg's Principle in the following manner (consistent with the modern view of Capek, 1961):

> Since either position or velocity may be fixed at choice, leaving the element of indeterminacy on the other side, both of them are shown to be conceptual in nature. That is, they belong to our intellectual apparatus for dealing with antecedent existence,

> not to fixed properties of that existence. An isolation of a particle for measurement is essentially a device for regulation of subsequent perceptual experience. (Dewey, 1929, pp. 202-203)

The notion of regulating subsequent perceptual experience for experimental purposes is similar to the theme of a recent book Behavior: The Control of Perception by Powers (1973). His contribution to this volume explicates more fully the constructivist orientation of his cybernetical analysis of behavior. (cf. also the contributions to this volume by von Glasersfeld and Silverman.)

Piaget and Dewey, Similarities and Differences

Similarities between the theories of Dewey and Piaget have been listed by Kohlberg (1969), and described as assumptions of the cognitive developmental model. First, it should be made clear that these assumptions have been derived from experiment and confirmed in experiment; they are not assumptions in the sense that they have been arbitrarily chosen. Second, before listing some of the similarities, I would like to call attention to the loose manner in which the word "environment" is used by Kohlberg (1969), a sense in which it is clearly possible to speak about the interaction between the organism's structure and the structure of the environment. The philosophical laxity of many cognitive developmental psychologists, in this respect, is a matter for the next section.

Kohlberg's list of similarities has been adapted to 'fit' the position of this paper and included: 1) development involves transformations of structures which must be explained holistically; 2) development is the result of the processes of interaction between the organism's structure and the structure of the experienced environment; 3) cognitive structures (schemata and schemes) are always action structures irrespective of whether the action is a physical or a cognitive operation; 4) the direction of development is always toward greater equilibrium in the interaction between the organism and its experience of its environment; 5) the apparent emphasis on cognition is resolved by maintaining that the affective domain is parallel to the cognitive.

The historical basis for Kohlberg's list of similarities is clearly stated by Piaget (1970b), a portion of this book which was originally written in 1935.

> On the one hand the work of the pragmatists had revealed the role of action in the constitution of all mental operations, and of thought in particular; on the other, the science of mental development, or genetic psychology, had increased considerably in scope, particularly with the work of Stanley Hall and J. M. Baldwin. These two trends found their exact point of intersection in John Dewey,.... Piaget (1970b).

Some of Kohlberg's similarities will be familiar from the preceding account of Dewey's epistemology, and they are not independent as they all refer to the origins and nature of knowledge. Both Dewey and Piaget describe "knowledge of reality" as a personally constructed organization of the sense data attended to; an organization which is tested each time it is used and which becomes more adequate as a result of modification after such use. We have seen that there is a denial that this knowledge can be correctly described as a reflection of the reality outside the organism in Dewey's case, and that this position came about as the result of his reflections on the implications of operationism, Peirce's pragmatism, and the Principle of Uncertainty. In Piaget's case there is a corresponding denial that knowledge is a reflection of reality; and an assertion that knowledge results from the organism's activities. The limited nature of the concepts which we construct to relate to our environment, in Piaget's theory, is held to be vindicated by Gödel's theorem. This theorem states that formal systems, which are at least as complex as arithmetic, are necessarily incomplete. Completeness is proven when it can be shown within the system that no logically incompatible statements can coexist within the system. What Gödel's theorem implied, and Gentzen later proved, was that in order to demonstrate completeness in any formal system one had to use principles of reasoning from a more complex formal system (Piaget, 1970c, p. 32). Piaget's inference is that knowledge itself can no longer be considered complete, since each new refinement will require justification at a newer and higher level. Smock (1973) has noted that, in part, this incompleteness stems from Piaget's recognition of the implications which the Theory of Relativity holds for psychological theory.

However, the recognition of this type of limitation does not impede the continuing construction of more adequate concepts, with which to relate to our environments, through the systematic application of the scientific method. This notion is as thoroughly Piagetian as it is characteristic of Dewey. In Piaget's theory conceptual development comes about through the equilibration process, which is adaptive, self-regulated, and dialectical. In other terms what is implied is that one's understanding of an experienced situation is limited by one's current understanding; new events are assimilated by existing structures. Aspects of these environmental situations which are different from current understandings of them, but within a match-mismatch range (Hunt, 1961) will be recorded by means of accommodations to the existing structures. The parallel with Dewey's account of experiencing is striking, especially the dialectical nature of experiencing in both theories.

Dewey (e.g., 1934) described experiences in terms of interactions between any live creature and its experience of its environment. These interactions consist of doing something and in consequence undergoing something. What is done and undergone will vary depending on the content of the experience, whether it is lifting a stone and assessing its suitability for some purpose, or selecting an appropriate statistical technique and checking that the assumptions aren't being violated, given the nature of the data. In this context the function of intelligence is defined by Dewey to be that of perceiving the relation between what is done and what is undergone as a result of a particular action. This action process of doing and undergoing continues until there is a mutual adaptation of the conceived ideas and the perceived object, and the experience comes to a close. Thus, when something is finished in a satisfying way we are prone to say that we have had "an experience" because what has been done and undergone has been completed and the result has coherence.

A consideration of the levels of possible operations in the two theories shows that Piaget's sensorimotor, concrete, and formal operations have parallels in Dewey's theorizing. Dewey's (1929) discussion of formal logic as a system of operations which act on other symbolic operations (p. 154) is clearly identical to one way of describing Piaget's distinction between formal and concrete operations; and Dewey's discussion of eye-hand coordinations (1933, p. 206) is a good outline of the carefully detailed developmental-observational studies of this phenomenon which were performed by Piaget. Other parallels which could be mentioned are Dewey's separation of thought and language, stressing the primacy of operations (1933, p. 183); and Dewey's analysis of play as activity pursued for its own sake (1933, p. 212; first published in 1910).

There are two aspects of Piaget's theory which are different from Dewey's theory: (a) the distinctiveness of figurative and operative processes, and

(b) Piaget's structuralism. In reference to structuralism, Dewey made no attempt to try to discover an underlying logic of the process of coordination of the cognitive operations. In Piaget's theory, figurative and operative structures have different origins and coordinate experienced sense data at different levels. Figurative structures originate in sensorimotor schemata; and operative structures, which originate in the schemes of the concrete operational period, coordinate the figurative structures. Piaget (1969, 1970a) has discussed the difference between the two types of structures in terms of the intentional transformational quality of operative coordinations. Thus, figurative cognitive functioning, which includes perception, imitation, and imagery, does not transform or seek to transform sense experience actively. The function of figurative operations being to represent "reality" by means of past organization of sense experience.

It is very difficult to eliminate the possibility of misinterpretation; perhaps it is impossible. The ambiguity with which Piaget obscures his epistemology has been pointed out by von Glasersfeld (1974) and is very clearly ambiguous in the discussion of figurative cognitive functioning (Piaget, 1970a, p. 717). When Piaget addresses representation of reality in figurative operations, it is very easy to understand him to mean that the structure of the environment matches the structure of cognition (cf. Smock's contribution to this volume). This position would imply an empiricist epistemology, and as such would be quite inconsistent with other parts of his theory. The constructivist epistemology, in both Piaget's theory and Dewey's philosophy, implies that what one knows about reality 'out there' is a personal construction. It is a personal construction which we can communicate effectively with other members of our species. However, it is a personal organization which cannot be described as an objective reflection of reality.

Interpretive Issues

The potential for misinterpreting, or not being aware of, the implications of the constructivist epistemology is obvious; and such misinterpretations are, I believe, quite general. One example mentioned earlier is Kohlberg (1969) who described development as the result of processes of interaction between the organism's structure and the structure of the environment. Similar laxity is found in a recent cognitive developmental account of the literature on imitation theory and research, Kuhn (1973). Kuhn referred to and adopted Kohlberg's (1969) idea of the environment. Again, Riegel (1973a) argued for an extension of Piaget's theory to include the historico-cultural context of development. Riegel's position was that Piaget's theory, at least in the 50's, overemphasizes the individual's role in cognitive development and underemphasizes the social context of development. For example, Riegel stated: "Development procedes through dialectical interactions between psychic activities and their biological and outer sociological interactions" Riegel (1973a). In each of these cases, the environment is interpreted as, or referred to as having an organization independent of the subject. However, psychologists are in good company when being loose philosophically because Piaget himself is treacherously ambiguous in his usage of the term "environment." For another example of this ambiguity, consider the following recent statement by Piaget (1972) on the relation between the subject and the object:

> In short, these pages contain an account of an epistemology that is naturalist without being positivist; that draws attention to the activity of the subject without being idealist; that equally

> *bases itself on the object, which it considers as a limit (therefore existing independently of us, but never completely reached)*; and that above all sees knowledge as a continuous construction:... (Piaget, 1972, p. 17). (my italics)

This unqualified reference to the "object" is confusing; however, I think that it is important to note that this quote is consistent with the view that while the "object" is independent of us, our knowledge of the object can only be described as the result of our intentional interactions with our experience with it.

Another important interpretive issue concerns the need for a final period of dialectical operations (Riegel, 1973b). This argument finds in the development of increasingly adequate logical competence a corresponding alienation from dialectical experience. Riegel qualified this interpretation as being true for Piaget's "middle period," that is his theory as represented by *The Psychology of Intelligence*. This present paper has emphasized the identity of cognitive process in Dewey's philosophy and Piaget's theory. Therefore, it is most important to point out that the dialectical nature of the theory derives from the dialectical nature of the equilibration process. So development is described in terms of reaching ever more adequate forms of equilibrium between cognitive structure and the individual's experience (which assimilates this experience in terms of existing structure). To argue that the dialectical nature of experience fades away as the structures become more logical is misleading. For example, Piaget (1968) described the equilibrium of the developing structures in middle and late childhood as more logical in the sense that they are more mobile. That is, these structures (of concrete and formal operations) emerge because they are better at resolving the dialectical interpenetration between the ongoing experience of the child and the existing cognitive structures. It is important to note that this discussion (the first article in Six Psychological Studies) was originally published in 1940 and so is representative of Piaget's "middle period."

In summary, a recent paper by Wohlwill (1973) has stressed the importance of recognizing, and taking steps to correct, the loose usage of the words "environment" and "experience" by psychologists. It is hoped that this present paper will contribute to a more general recognition of the peculiar status of "the world" within the constructivist epistemology, because it is this odd status which is the major contribution of this theory to epistemology. Within this present context, it is only possible to refer to knowledge of the environment, or experience of the environment, in the context of any particular individual's experience. To neglect this emphasis is to miss a crucial aspect of the constructivist position. That it seems so easy to neglect is perhaps due to current psychology's relative neglect of Dewey, who did emphasize this, and relative preoccupation with Piaget who seems unwilling to emphasize this.

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Epistemology and Mathematical Proof

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> Mathematics can achieve nothing by concepts alone but hastens at once to intuition, in which it considers the concept *in concreto*, though not empirically, but only in an intuition which it presents *a priori*, that is, which it has constructed,...
>
> Immanuel Kant
> *Critique of Pure Reason* A715-716

Introduction

1930 was a very good year for philosophers looking at mathematics. A clear case of a philosophic issue -- the nature of mathematical truth -- divided mathematicians into three schools of thought: Logicists, Formalists, and Intuitionists. But by 1935 the nice division was a wreck! For Logicists, mathematics was to be derivable within logic, but the self-referential paradoxes and the general uneasiness about the Theory of Types had ended the initial excitement over Logicism. Intuitionism had made great strides in pointing out shortcomings in classical mathematics, but the restriction to a constructive mathematics was not a reasonable alternative for the working mathematician. And Formalist hopes of proving consistency of a system within that system were left shattered by Gödel's Incompleteness results.

Since then philosophers have been avoiding certain problems in the foundations of mathematics. The basic foundational programs of the early 20th century were epistemological in nature. Recent philosophical inquiry into mathematics has focused on ontological issues ("What *is* X?") and not on epistemological issues ("How do I know X?" or "What is the X I know?").

I would like to thank John Messer, Don Nute, Ernst von Glasersfeld, Hugh Gash, and Leslie Steffe for comments on earlier versions.

As a result, there has been an overemphasis of the differences of the above three "schools." Meanwhile, logicians with a philosophical background have turned towards problems of language, linguistics, and formal languages. This is unfortunate, since many of the recent advances in logic have crystallized new positions in foundational studies in mathematics. Today there are clear philosophical questions and alternatives.

Perhaps the most important item learned from the philosophy of mathematics in the 1920's is that there is no single CORRECT foundation of mathematics. Rather, there are series of more or less acceptable alternatives depending on your conception of mathematics -- and on what you want to prove. The paper will center around the "constructivist" tradition in foundational studies of mathematics. This tradition has focused on mathematical activity, and has sought to make it reliable.

The uniqueness of mathematical activity lies in the peculiar status of mathematical knowledge. Thus our concern will be to provide a foundation for a theory of mathematical knowledge or more appropriately mathematical epistemology. The first goal of this paper is to show the mutual compatibility of the Formalist, Intuitionist and Logicist programs. If we view them as approaches to analyzing mathematical knowledge, they are not only compatible but in fact complementary.

The second goal of this paper is programmatic. Suggestions are offered for a radically different approach to philosophy of mathematics from what is now popular among philosophers. I argue that a thorough investigation of mathematical epistemology is sorely needed, and should supersede the current trend towards mathematical ontology. The basis for this approach lies in recognizing mathematics as a human activity. In so doing the act of discovery can be separated from the communication of the discovery; what Georg Kreisel (1971) has called a separation of "proof" from "derivation." With this central dichotomy in mind the positions of Logicism, Formalism and Intuitionism are re-examined. In addition, the problems and possibilities involved in making this dichotomy more precise are examined.

The approach in this paper is in many ways broader than traditional philosophy of mathematics. The Formalist and Intuitionist programs were concerned with the progress of mathematics as a field. They focused on the behavior of an ideal mathematician. They were concerned with a total reconstruction of mathematics in order to place it on firmer footing. I want to propose a general critique of the human activity called mathematics. In order to accomplish this, I too will focus on the "ideal" mathematician -- the progress of mathematics as a whole. But I will also focus on the "bad" parts of mathematics -- the mathematical activity the rest of us engage in. This will bring us to a notion of construction which is different from that of the Intuitionists and different from that of the Formalists (and of their descendents, the Proof Theorists). The essence of this difference is in the dynamic character of construction, a process, as opposed to the static form of construction, an object, used in current mathematical discourse.

Logicism, Formalism, and Intuitionism -- The Ontological Views

Kant asserted that mathematical knowledge was synthetic *a priori*. It was knowledge gained prior to experience, but in mathematical statements the predicate was not contained in the subject. That is, for Kant mathematical statements were empirical, they tell us something new. Bertrand Russell and Gottlob Frege reacted against Kant, and Logicism was the position that mathematical knowledge was analytic *a priori*. They agreed with Kant that mathematical knowledge was gained prior to experience but, for them, mathematical statements were not empirical and did not give us any new information. In order to prove this, it would suffice to show that mathematics was a part of logic, or rather, that all mathematical truths were derivable from the truths of logic, and that all mathematical concepts were reducible to logical concepts.

The motivation for both the Intuitionists and the Formalists was much more practical and immediate. They were mathematicians reacting to a foundational crisis in their subject matter. In the latter part of the 19th century mathematicians had felt secure in their work. The "Queen of the Sciences" was whole and virtuous. But explicit contradictions arose in previously "secure" areas, and in losing its virginity (for the third time) mathematics was scarred. Roughly there were three reactions.

(a) The constructive tendency, resulting in Intuitionism, attributed the contradictions to the use of "negative" results in proofs. In order to prove that there exists something with a particular property, it is necessary to produce it. The indirect proof which showed that its non-existence led to contradictions (reductio ad absurdum) was not sufficient. As mentioned earlier, the interest was in mathematics as a human activity dependent on human ability.

(b) The Formalist reaction to the appearance of contradictions was to emphasize the importance of consistency proofs. If we construct a formal language and can prove that it is consistent then we can feel secure within the confines of that language. Mathematical activity is essentially concerned with formal languages, and with the obvious manifestations of formal systems, namely signs. Formalism was a working philosophy for the geometrician in the latter part of the 19th century, and was made explicit as a result of its success.

(c) "Normal Science" proceeded unaffected by the crisis.[1] Each mathematician was confident that his particular specialty was on firm ground. Although there might be paradoxes (not contradictions) lurking around the corner, as long as contradictions were not appearing in the proofs, the proofs were acceptable.

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[1] The term "Normal Science" is adopted from Thomas S. Kuhn (especially The Structure of Scientific Revolutions volume 2, number 2, International Encyclopedia of Unified Science 2nd edition, 1970, University of Chicago Press, Illinois). It is used here to characterize the activity of non-foundational mathematics, and to emphasize the "working mathematicians" activities and attitudes.

The tripartite division between Logicists, Formalists and Intuitionists was fairly rigid and quite amenable to easy analysis. But the problems with this easy analysis lay in the failure of philosophers to examine (1) the original intent of the programs or (2) what I have called "normal science." They assumed that each of the schools addressed a single question and that the divergent answers each produced indicated three distinct points of view. The division seemed to be very close to the ancient philosophical debate, viz. the nature of reality. The question therefore appeared to be, "What is the nature of mathematical reality?" or more aptly, "What is the nature of mathematical entities?" The emphasis of contemporary study in the philosophy of mathematics therefore centers around the question, "What is the nature of number?", which may help to explain its irrelevance to the working mathematician.

Due mostly to Frege and Russell, the Logicist claims that numbers are real objects which are "out there." "Numbers, the Homeric gods, relations, chimeras and four-dimensional space all have being, for if they are not entities of a kind, we could make no propositions about them." (Russell, 1903, p. 449). The Formalist recognizes the dependence of mathematics on language, and argues that mathematics is a theory of symbols, having no relation to real objects of any sort. "...[I]n mathematics, in particular, what we consider are the concrete signs themselves, whose shape, according to the conception we have adopted is immediately clear and recognizable" (Hilbert, 1967, p. 376). And again, "All the propositions that constitute mathematics are converted into formula, so that mathematics proper becomes an inventory of formulas." (Hilbert, 1967, p. 465). The Intuitionist recognizes the peculiarly human aspect of mathematics, and concludes that the existence of mathematical entities is dependent on the mathematician. "...We do not attribute an existence independent of our thought, i.e., a transcendental existence, to the integers or to any other mathematical objects...Even if they should be independent of individual acts of thought, mathematical objects are by their very nature dependent on human thought." (Heyting 1964, p. 42.)

Thus we have a nice division. Logicists are Realists who believe that numbers are real objects. Formalists are Nominalists who believe that numbers are not real objects. And Intuitionists are Conceptualists who believe that numbers are products of our thought.

We find ourselves marvelling at the accuracy of the medieval division into Realists, Nominalists, and Conceptualists. Rather than learning from the frustrations of the last 500 years of ontology, we throw ourselves into the timeworn debate. In this instance we apply blinders and only observe numbers. This over-simplified account is still applicable especially to philosophers looking at mathematics, as opposed to mathematicians looking at foundational questions. It is important to make this distinction.

This ontological question about the way in which a number exists dominates the recent history of the philosophy of mathematics. The answer always centers on our knowledge of the nature of number. Instead of dealing with these epistemological results and analyzing them, we get a perversion of the analysis in an attempt to answer the ontological problem.

The results of mathematicians looking at the foundations of mathematics have been much more fruitful. The emphasis has turned, over the last thirty years, to an analysis of the notion of proofs. That is, the central question has been of the sort, "What kind of mathematical knowledge can I depend on?" or alternatively, "How can I preserve rigor in mathematics?" But these questions are not really new, nor is the concern with the nature and reliability of mathematical activity new. Logicism, Formalism, and Intuitionism were all concerned with problems of mathematical knowledge.

For the Logicist, the central concern was the status of mathematical truth. For the Formalist, the central concern was the consistency of mathematical systems. For the Intuitionist, the central concern was the reliability of mathematical activity. Although we may shift our attention to the nature of mathematical entities, and provide ontological counterparts to each of the programs, there is no benefit in proceeding in that manner. In addition, we lose sight of the original programs and forfeit many of their positive contributions. In the next section, I examine various aspects of mathematical activity in general, and formulate several distinctions which will facilitate our re-examination of the three "schools" in the philosophy of mathematics.

Mathematical Activity --Preliminaries

Mathematical activity is not unique, but it provides clear examples of distinctions which occur elsewhere, but in more obscure contexts. Although mathematics is the object of our present study, we could alter some of the more specialized terminology and arrive at a general development of the way we proceed in any abstract study. Much of what we say will be applicable to any scientific discipline, but that is beyond the scope of the present paper.

I will begin my discussion of mathematical activity by laying out some assumptions. First, I assume that there is non-linguistic thought. Second, this thought is non-propositional. It may not be expressed in the form of propositions, although, obviously, propositions (such as these) may be about non-linguistic thought. At best, propositional language may "show" or "point to" this kind of thought. It may not literally express it. Third, non-linguistic thought is not expressible in/by any language. This holds in principle, and does not seem to be a limitation of our present languages.

We develop more powerful languages by allowing our non-linguistic insight to expand in a boot-strapping effect. Fourth, it is objective, in the sense of being inter-subjectively testable. This is the weakest of the claims, and the only one about which I am hesitant. I will try to make it more explicit and more appealing at a later point in the paper. And fifth, I assume that this thought is central to doing mathematics, as opposed to describing mathematical acitvity. The latter is clearly propositional in nature, and is what I am engaged in here.

Non-linguistic thought is very often vague and can be self-contradictory. This is not to say that it is devoid of meaning. A self-contradictory proposition "...does imply, or mean, something. A self-contradictory proposition is not meaningless; it means too much, i.e., it means both p and not-p."(Peirce, 1932, Sec. 2.352, p. 201). Similarly, self-contradictory thought is not devoid of meaning. Rather, it accepts too much, and certain lines of development must be rejected. We formulate an idea roughly, and in our attempt to describe it (prove theorems about it?) we make it more precise. It is this process which has led mathematicians to regard themselves as "Realists" or "Platonists." We "see" an object and attempt to describe it. Noting the objective character of the description, we get the impression that there is something "out there" that we have glimpsed. Our mathematical activity, then, consists in describing what we have seen.

I propose that this is a misdescription, and that the ontological questions are superfluous. We construct an image ourselves. It is not as if we peek behind a screen. Mathematics AS WE KNOW IT is a peculiarly human activity. It is an attempt to deal with the process of generalization or abstraction. The question of the "reality" of our imageing (sic) or our images is irrelevant. The emphasis is to examine the mathematical enterprise as a human activity based on human cognition. It is fundamentally epistemological. Our description of it must be ontologically neutral. We are not concerned with what we are examining, an elusive goal at best, but with how we create mathematics, and consequently, how we describe our creations.

We often speak of how we perceive numbers and due to our dependence on the sense of vision we constantly use visual metaphors in mathematics. The only exception I can think of is that we sometimes say we "smell" a proof, indicating that we "feel" that the proof is possible but it is not yet precise. What would it be like to hear a proof or taste it? What kind of abstraction would obtain from senses we do not have at all? What kind of mathematics would be done by non-human creatures and how would it differ significantly from what we know as mathematics?

Mathematics is a product of human effort. The key is not to analyze the nature of number, but to analyze our conception of number. It is essential that we distinguish this from a (passive) perception of number. The notion of a conception of number emphasized the dependence of mathematics on humans, but an analysis of a perception of number leaves open the nature of number apart from our perception of it. More importantly if we speak of a perception, it leaves the impression that there are numbers existing "out there." This impression leads to the question, "What are numbers really like, apart from our perception of them?" and so on.

By restricting our enterprise to an analysis of our conception of number we reemphasize the strictly "human" character of mathematics. The emphasis is on the nature of mathematical reasoning, not on mathematics as some abstract entity lying completed under some unturned rock, or in some absolute mind. The emphasis ought to be, as stated above, on how we create mathematics, and consequently, on how we describe our creations.

These two processes, creation and description, are different and must be examined separately. Frege states, "... We can inquire, on the one hand, how we have gradually arrived at a given proposition and, on the other, how we can finally provide it with the most secure foundation. The first question may have to be answered differently for different persons; the second is more definite." (Frege, 1967, p. 5). We have to distinguish between what Dewey called a "logic of discovery" and a "reconstructed logic." Frege's Begriffsschrift was a reconstructed logic. Frege's system was a definite, describable object. The problem with the logic of discovery is deeper than the fact that it may be different for different people. Rather, it involves pre-linguistic thought. It is inherently not describable by language. Bourbaki, (1949, p. 1) makes a similar claim with respect to mathematics: "Proofs, however, had to exist before the structure of a proof could be logically analyzed. ...Logic, so far as we mathematicians are concerned, is no more and no less than the grammar of the language which we use, a language which had to exist before the grammar could be constructed."

It is absolutely imperative to distinguish between mathematical insight which is an act of discovery, and the careful recapitulation or recording of this insight. The insight is an action of an individual. This is not unique to great mathematicians, or even good ones. We all have a feeling of "aha" when we "see" that a proof succeeds, or when we "see" that we are coming close to a result.

Insight is an individual, personal, inherently subjective experience. It does not apply to commonplace knowledge but only applies when we are dealing with the "fringe" areas of our knowledge. We do not need insight to "see" that 2x2=4 or that 7x8=56, although a child might. For each person insight is applied to the frontiers of his knowledge. By extension, in mathematics, insight by good mathematicians leads to new results.

Kreisel distinguishes between a "proof" and "derivation." A proof involves insight and is in the category of what I have called the "logic of discovery." Kreisel claims that proofs are intensional objects, explaining this partially in a footnote. "Following current terminology I use 'express' for the relation between a formal expression E and the intensional object meant by E,..." (Kreisel, 1971, p. 242). Thus, proofs are "expressed by formal derivations." Derivations are, in some sense, the recordings of proofs. Derivations will fall in the category of "reconstructed logic."

When we communicate a proof, it is common to say, "Let me reconstruct it for you." Here it is clear we are using a reconstructed logic to attempt to communicate an abstract object. We use reconstructed logic to make proofs objective and not subjective. The communication and repetition of these abstract structures is the central issue of this paper. Proofs often involve reference to images or intuitions. Further, much of the "pre-proof" activity we engage in is solely concerned with "picturing" a mathematical object and attempting to describe it. This is the specific non-linguistic activity to which we alluded earlier. This activity is by its very nature imprecise. It is a start towards abstraction and generalization. It grows out of a "feeling" that there is (or ought to be) a connection between previously unrelated data.

This kind of discussion comes dangerously close to psychologism which Frege attacked so vehemently, and perhaps a word of explanation is in order. It is clear that a discussion of mathematical activity should not appeal solely to introspection. But our mathematical intuitions and images are not always clear and distinct. They are often vague, and they often lead to contradictions. The positions we consider or image are often not subject to a more careful analysis. We approach a problem and consider, "What if...?" and try to imagine a structure operating in a certain way. But how do we do this? To begin with, we try to make it more precise. We see if it is consistent with other structures. We see if some standard theorems, which we expect to be able to prove, are provable. We try to turn or twist a topological structure to examine it and look at the properties of the contorted figure.

The relationship between a proof and a derivation is left somewhat obscure by Kreisel. For Kreisel, proofs are expressed by derivations, and are the meanings of derivations. He is mistaken. Proofs can only in a very loose sense of the word be expressed by derivations. A derivation entices, or leads us into seeing a proof. It does not tell us (communicate directly) the proof. G. Spencer Brown explains mathematical communication in a similar fashion.

> It is comparable with practical art forms like cookery, in which the taste of a cake, although literally indescribable can be conveyed to a reader in the form of a set of injunctions called a recipe. Music is a similar art form, the composer does not even attempt to describe the set of sounds he has in mind, much less the set of feelings occasioned through them, but writes down a set of commands which if they are obeyed by the reader, can result in a reproduction, to the reader, of the composer's original experience. (Brown, 1969, p. 77)

The set of injunctions is the derivation. The taste of a cake, or the sound of a symphony, or the awareness of a proof are all similarly indescribable.

Proofs are inherently subjective, as is insight which forms a significant part of proofs. They attain a degree of objectivity only insofar as we agree that a particular derivation has led us to see a proof. We often get the meaning of a derivation prior to seeing the proof. Roughly, although not always, the construction of the proof corresponds to the "logic of discovery" and the derivation corresponds to the "reconstructed logic." This is not completely accurate, because the construction of the derivation may involve the logic of discovery.

Proofs provide the motivation for doing derivations, they are not literally the meanings or intensions of derivations, although they are intensional objects. Kreisel does not make it clear in what sense proofs are intensional objects, nor, for that matter, does he explain what an intensional object is. In his review of a recent article by Kreisel, John Corcoran points out that it is common practice to distinguish "...[B]etween proofs as intentional objects and derivations as syntactic objects (= types not tokens) which may be said to represent or express proofs...No journal would publish two derivations which express the same proof but a journal may well publish two different proofs of the same theorem." (Corcoran, 1972, p. 712). It is interesting to note that Corcoran refers to them here as intentional (with a "t") objects. The distinction is an important one. Intension (as opposed to extension) is generally used as synonymous with the connotation or sense of a term. It refers to a property or attribute, rather than the class of objects satisfying the property or attribute (which is the extension, denotation, or reference). Proofs are intensional objects in the general sense that they have the same kind of status as attributes or properties. Intentional objects are intended or purposeful, indicating an important relationship between the object and the "creator," i.e., the individual intending. This also stresses an important aspect of proofs, namely that they are always with purpose. We must always have a proof of something. It is helpful to recognize that proofs are both intensional and intentional.

The reason for doing derivations is very clear; to try to show proofs, to get someone else to see a proof. But derivations are rarely, if ever, evaluated along these lines. We can better understand that use of derivations if we examine what they ought to do. (1) Perhaps the most important aspect is that derivations render proofs intersubjectively testable. This may help to explain the objective character of some of the "pre-linguistic" thought mentioned earlier. (2) Derivations permit a careful analysis of proofs. This allows us to distinguish between (a) false hunches, (b) good guesses (for the wrong reasons) and (c) insight, a necessary but not sufficient condition of which is, good guesses for the right reasons. (3) Derivations are useful in making vague concepts more precise. We may have a rough idea of how a proof ought to proceed, but until we actually do a derivation of it, we are not clear on many of the details.

It will be helpful at this point if we introduce an additional distinction which has not previously been made in the literature, viz between a "derivation" and a "deduction." A "derivation" will be any symbolic representation of a proof. A "derivation" may make use of pictures, diagrams, written and spoken symbols, and a certain amount of hand waving. A "deduction" will be a syntactic mathematical object which occurs in formal languages. Typically it is a sequence of well formed formulas of the language satisfying specified conditions, viz. that each is an axiom or obtained from previous expressions in the sequence by rules of inference.[2] The definition of a deduction is not made more precise here because it is dependent on the pecularities of the formal language for which is is defined. It is clear that there are many deductions which are not derivations of any proof. This occurs especially in logistic type axiomatic systems, where each line of a deduction could be viewed as trivially the end of a deduction (cf. J Corcoran, 1969).

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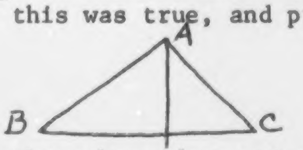
[2] Consider Euclid's assertion: If through the vertex A of a triangle, a ray is drawn between the adjacent sides, then it will intersect the third side. Intuitively Euclid saw that this was true, and provided the following derivation:



but it is important to note that there does not exist a deduction of this from Euclid's axioms -- we need additional Axioms of Continuity to insure that the ray from A, and BC have a point in common.

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This distinction will be helpful in separating the logician's viewpoint from the mathematicians'. Specifically, since most of the people looking at foundations of mathematics (the present author included) have had their training in logic, there is a tendency to view the extreme formalization of logical systems as typical. In Corcoran's review we have the following discussion:

> In connection with the proposition that formal derivations express intuitive proofs it is worth noting [that] ... some authors (e.g., P. Gilmore [1960]) do not regard formal derivations as related in any way to informal reasoning but rather they think of a formal system of derivations as providing a means of mechanically producing from a set of sentences its logical consequences. (Corcoran, 1972, p. 712)

This is seen as a general argument for Natural Deductive systems as opposed to Axiomatic Systems. In fact, most derivations (in my sense of the word) are not carried out in formalized languages, but rather in informal naive (in the sense of "naive set theory") languages.

The implications of this are extreme. Although the above reasoning will most likely lead to (and in fact has led to) a "concentration on construction of increasingly rich systems of natural deduction..." (Ibid) the effects on other areas of mathematics are more severe. The reliability of mathematical reasoning may, in the end, revert to the logical underpinnings, but this is not the questionable portion of a mathematical proof. "When we really want to make sure that a mathematical result is correct, we do not formalize its proof, i.e., compare the steps with given formal rules, but we try to make it intelligible. We rely on mathematical, not logical foundations." (Kreisel, 1968, p. 361). But this should not be true in principle, and, more than anything else, points out the inadequacy of the logic in use. This realization should lead to a more useful derivation technique, or, at least, it indicates a need for a theory of derivation technique. It is clear that Natural Deductive systems are a closer representation of our logical thought processes than Axiomatic systems, but how can we make this observation precise? (c.f. J. Corcoran, 1969).

The Epistemological Approach to Philosophy of Mathematics

We can utilize the distinctions in the previous section to characterize various aspects of mathematical activity. The programs discussed in the first section can be best understood if we see them as treating separate aspects of mathematical activity. The insight gained by each of these approaches turns out to be not only compatible (they do not contradict each other), but in fact, as we will show in a later section, they are complementary (they serve to support each other).

The first of the programs I want to review is the Frege-Russell construction of mathematics out of logic. They wanted to show that the truths of mathematics were derivable from the laws of logic. But if mathematics is a human enterprise, it should be a result of human logic. Logic for Frege and Russell included set theory, but it also was concerned with the "laws of thought." If we accept that all mathematical concepts are expressible in (reducible to) that part of mathematics we call set theory, then if we embed this in our patterns of thought we ought to come up with all of mathematics, or more precisely, all of our conception of mathematics.

The question then is, "Are all mathematical concepts expressible in (reducible to) set theory?" This is really two questions. It is possible that all mathematical concepts are expressible in set theory, although I doubt it. But it is not even clear how we would judge that all mathematical concepts are reducible to set theory. Further, by elaborating on the question of expressibility, we come up against the problem of pre-linguistic mathematical thought. Thus, we might revise the question to, (*)Are all statements containing references to mathematical concepts equivalent to

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statements containing references only to sets? We specifically avoid the "logic of discovery" by limiting our enquiry to statements. Frege's system "...[I]s a device invented for certain scientific purposes, and one must not condemn it because it is not suited to others. If it answers to these purposes in some degree, one should not mind the fact that there are not new truths in my work." (Frege, 1967, p. 6).

Thus, if we view the Logicist claim as restricted to a reconstructed logic, it becomes plausible On the propositional level, (*) has a good chance of being answered positively. Specifically, consider the following questions, "Are numbers reducible to sets of sets?" We want to be careful about the way we address this. We do not want to reword the question, "Do we conceive of numbers as sets of sets?" or genetically, "Do we learn about numbers by generalizing about sets of objects?" Rather we want to ask, "Are statements about numbers reducible to statements about sets?" If the answer is positive, then the Russell-Frege program ought to follow trivially from our understanding of reconstructed mathematics. Note also that the question of whether mathematical concepts are reducible to set theoretical and logical concepts is a question about the logic of discovery, i.e., a question of how we conceive of mathematics. I will return to it later.

In light of the distinction between "reconstructed logic" and a "logic of discovery" we can also see clearly the source of some of the major disagreements between Brouwer and Hilbert. Brouwer explains, "The question where mathematical exactness does exist, is answered differently by the two sides; the intuitionist says: in the human intellect, the formalist says: on paper." (Brouwer, 1964, p. 67). If we view Brouwer as attempting to explain a portion of the logic of discovery, and Hilbert as dealing with reconstructed logic, then we see that their subject matter is different and it should not be surprising that the answers differ. I intend to show that there is nothing contradictory about maintaining that both sides are correct.

Brouwer, in his short commentary on some of the similarities between the Formalist and Intuitionist programs, makes a similar distinction within mathematical activity, and (correctly) notes the compatibility between the two programs in light of "The differentiation among the formalistic endeavors, between a construction of the 'inventory of mathematical formulas' (formalist view of mathematics) and an intuitive (contentual) theory of the laws of this construction..." (Brouwer, 1967, pp. 490-1). The "inventory of mathematical formulas" taken by itself is useless; it requires a motivating factor, which is provided by the "intuitive theory." Again this division corresponds to the division between the "reconstructed logic" and the "Logic of discovery." It also marks a distinction in the mathematical activity between doing "derivations" and doing "proofs." Brouwer's distinction, I believe, may emphasize the wrong aspect of the division. "Derivations" have form but are clearly not without content entirely. Conversely, "proofs" have content but are not devoid of form. Kreisel has recently argued for a similar point. "The data of foundations consists of the mathematical experience of the working mathematician...The processes involved in mathematical activity present themselves as primarily mental and abstract (not sensory), but they are recorded and communicated by use of CONCRETE, i.e., spatiotemporal, symbols." (Kreisel, 1965, p. 95).

For the Formalist the emphasis of his study, and the resultant data, is the recording and the communicating, and therefore mathematical activity is symbolic. For the Intuitionist the data is the activity. His emphasis is on the abstractions, and the construction of the mental processes. The importance of viewing this division epistemologically and not ontologically is that there is not a choice between the nature of mathematical objects, but rather a choice of various aspects of mathematical activity. It is clearly too restrictive to conclude that either part of the activity is more important, or more central, than the other.

In short, we must take into account the distinction between DOING or LEARNING or CREATING mathematics, and RECONSTRUCTING, or REITERATING, or TEACHING mathematics.[3] The former requires an intuition, an image, the latter does not require it, and in fact (as with Euclid) it may prove a distraction.

The question of the limits of visualizability is central to understanding the implications of this distinction. Hilbert argued that any construction, including those we visualize, must have a finite structure. In support of this, it is clear that a formal deduction must be finite. We cannot write an infinite number of symbols. This is very different from saying that we cannot write symbols which stand for infinite objects.

If the object of mathematical study is solely the spatiotemporal objects which constitute our recapitulation of proofs, then mathematics is necessarily finite. Note the stress on the real world which is essential for Hilbert. "And the net result is, certainly, that we do not find anywhere in reality a homogenous continuum that permits of continued division and hence would realize the infinite in the small. The infinite divisibility of a continuum is an operation that is refuted by our observations of nature and by the experience gained in physics and chemistry " (Hilbert, 1967, p. 371). The mere fact that we can only imagine it is not sufficient to guarantee existence for Hilbert. However, if we view mathematics from an epistemological perspective, we see that Hilbert is again not disagreeing with Brouwer.

No one denies that we have the ability to write only finite numbers or symbols. But similarly, neither Hilbert nor Brouwer denies that we can conceive, think about, visualize, imagine, or construct infinite sets of objects or infinite structures. It is interesting that we can redefine our notion of infinite to give it a more "acceptable" character. That is, we can define a set of objects as "infinite" if they have the property that they can be placed into a one-one correspondence with a proper subset. And we have ways to represent this within formal derivations.

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[3] I would argue though, that good teaching is defined as setting up conditions in which persons can discover, which emphasizes even more the need for an analysis of the "logic of discovery."

The restriction inherent in Hilbert's view can be made more obvious if we examine a statement of Max Black's about the nature of mathematics. ".... Mathematics is the study of all structures whose form can be expressed in symbols, it is the grammar of all symbolic systems and, as such, its methods are peculiarly appropriate to the investigation of its own internal structure." (Black, 1949, p. 4). But this is wrong, as we have just intimated. Mathematics goes beyond symbols. If it were restricted to symbols, then mathematics would be restricted appropriately enough to finite structures, and it is not. Here again, we see that the terms "finite" and "infinite" are really misleading. It is not possible to write a set of symbols which can be put into a one-one correspondence with a proper subset, but it is possible to examine the other properties of sets which exhibit this particular property.

Constructions

Now let us return to the mathematical activity itself. Since we no longer need to view the various philosophies as competing, we can benefit from the independent insights of each of the "schools." The most important of these insights focuses on the reliance of the mathematician on constructions. The limits of constructability is a crucial question for both Intuitionists and Formalists.

The ordinary notion of construction has been abused by both of these programs. I will not hold ordinary language up as an ideal, but an examination of the notion of construction, as we commonly use it, will give us some insight into the logic of mathematical discovery, an epistemological pursuit.

There is an ambiguity in the use of "construction." It is used to refer to the activity, and to the result of the activity, of constructing.[4] When we see workmen engaged in construction, we say they are "constructing a building." (Notice also the odd use of "building" here to label the product.) This notion of construction as a process is virtually ignored in "mathematical construction." It was initially, in Greek mathematics, limited to ruler and compass diagrams. At the time this was all that a man could do precisely. But man's capabilities were underestimated. Not only was he able to extend the notion of precise construction in Euclidean geometry by the use of other accurate tools, but he was able to abstract from this process.[5]

In the future, new generalizations and techniques may allow us to construct objects which are not imaginable to us now. We might state this more accurately by saying that they are not imageable to us now. A new technique gives us the possibility to create, not discover, mathematical entities; and it gives us the possibility to discover, not create, relationships in the "real world." As we develop our tools, constructible by means of ruler and compass is seen as a purely arbitrary limitation.

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[4] As pointed out by Ernst von Glasersfeld (1974), the same ambiguity applies to "representation" in psychology.

[5] This development of the notion of "construction" from geometry may help to explain our reliance on visually oriented metaphors.

The process or functional definition of "construction" affects our inquiry in three ways. First, it emphasizes the need to take the growth of mathematics as a subject into account when we offer a characterization. In this way we view mathematics as a "living" subject matter, which is still in its infancy. Second, it emphasizes that mathematics is an activity, a process. People do mathematics. Third, as a result, it correctly directs our attention to a logic of discovery, to the way we form constructions.

The essential difference between this view and the static view of constructions is that both Intuitionists and Formalists viewed constructions as completed objects. It was then possible to examine the properties and limitations of these objects. Instead of examining objects, I want to examine the process of making these objects, with the distinct possibility that there are many constructions which are not yet completed or even started. The focus then turns to possibilities which are qualitatively different from those constructions we now can examine.

Through this approach we have a better chance of accounting for growth in mathematics. If we look at "construction" restricted to "being built with a ruler and compass," as in ancient Greece, then a discussion of the limitations of construction is necessarily incomplete. To a large extent we are dealing with an open-ended process. As we accept new and different ways to construct mathematical entities we enlarge our definition of construction, and coincidentally we expand the areas of application of mathematical knowledge. There is a vast amount of historical evidence to support this view of the growth of mathematics. Man has in fact expanded his ability to construct abstract objects.

Let us return again to the debate between Brouwer and Hilbert. For Brouwer, mathematics is an attempt to deal with human ideas, and, as both Hilbert and Brouwer recognize, these may include infinite structures. For

Hilbert, mathematics is an attempt to deal with signs and these are necessarily finite. The resolution is to view mathematics restricted to neither of these possibilities. This also applies to Brouwer's restrictions. He restricted the level of the infinite which we intuit as being the limit of our intuition of adding one, i.e., ω ("omega"). But here also we are only restricted by the ability of present mathematicians to feel at home with a particular concept. Although Brouwer focuses on mathematics as a human activity, he fails to take into account the expansion of our techniques of doing mathematics. He wanted the natural numbers to be secure, and the results of work with them to be dependable. Rather, I want any procedure a mathematician engages to be secure and dependable, with the clear possibility that he will formulate new procedures.

As we mentioned earlier, the progress of mathematics involves broadening the concept of constructibility. We are still able to expand our ability to imagine and we are still able to generalize on procedures currently in use. This extends the concept of constructibility in general, and its application to proof in particular. We need broader notions of constructibility in order to explain the possibility of growth. It is folly to examine the current state of mathematics and postulate that we are approaching any limits whatsoever.

This broader notion of constructibility benefits from Brouwer's focus on the law of the excluded middle. H. B. Curry, a formalist, finds correspondence between their views. "Thus the propositions which arise from a non-constructivist argument are not to be dismissed as *per se* meaningless. They have at least a heuristic value; they show what may result if something more is assumed; and in many cases they many be formalized and so give rise to purely mathematical propositions at a higher level." (Curry, 1950, pp. 56-57). If we are able to show (as in *reductio* reasoning) that the assumption of the negation of an existential proposition leads to contradictions, then we still are not entitled to assert the existence of the object. But we can use the information obtained to eliminate some possibilities and to guide future research.

A standard example of indirect reasoning which is unacceptable to the Intuitionists is the proof that the number of primes is infinite. The proof does not provide a construction, i. e., an algorithm for locating (or producing) them. Instead it show that the assumption that the number of primes is finite leads to contradicitions. What can we conclude from this? At the least we can, as Curry suggests, use it as a heuristic to guide future research (namely, if you look at the largest prime number we have found to this date "p", it would not be a useless enterprise to look for more). In fact we know (in an important sense of the word) that there is another prime between p and $p! + 1$.

As Brouwer has told us, indirect arguments expand our knowledge in a very limited fashion. When we assert that "x exists" as a result of indirect argument, we are not given an image of x. We are only told that it is possible to construct an image. The fact that we have not constructed it is a limitation of our knowledge at that time. These limitations may apply to all of us equally, in which case there is no known solution and we find ourselves at the frontiers of mathematical knowledge. Or they may occur, and in fact occur quite often, to students as they learn mathematics and quickly, in a matter of a few years, recreate the images formulated by generations. Aided by derivations and the benefit of hindsight, they overcome temporary setbacks.

As with the progress of mathematics as a field, so with the progress of a mathematician. It takes a great deal of time for the mathematical community to accept a new construction, technique, or way of imageing. Similarly we learn to visualize and we enter into a process, the end result of which may be that we "see" a proof, or a technique. This is rarely immediate, and in general requires a considerable amount of effort.

> Much of the most important and vital work done in the last half century depends upon new ideas, and new ideas are notoriously exceedingly difficult to grasp. When we first make their acquaintance they often appear so strange, so intangible and so remote from our normal mode of thought that we may be completely lost. All that we can do is to read and re-read, and in time the vague shadows, which seem to lack reality or even substance assume a definite form; the strangeness disappears through repeated contact and then we can begin to appreciate the new idea for what it is, its significance and power, and even the great simplicity it has introduced into many arguments. (Mordell, 1959, p. 11.)

This picture Professor Mordell paints is a careful recapitulation of part of the mathematical experience. The dependence on metaphor and poetic language is typical of many similar attempts. I might add that the way we come to accept a new idea is a gradual process. As Bernays points out, self-evidence is an acquired characteristic, greatly dependent upon our experience in the particular field (Bernays, 1954).

In Kreisel's discussion of the Hilbert program he states, "Logically, as is well known, the simple minded idea is that we want our proofs to be "reliable"; ...or quite explicitly (as Hilbert once said) that Zermelo's axiom of choice be... reliable in the same sense...as 2+2=4" (Kreisel, 1970, p. 492). But it is important to note the change in attitude that has occured towards the axiom of choice since Zermelo first stated it in the beginning of the Twentieth Century. The old tendency to place an asterisk before proofs which rely on the axiom of choice has, for the most part, been abandoned. What change has been brought about? More than the independence results of Cohen, there is a subtle change in common sense. The passage of time, the corresponding failure to arrive at contradictions, and perhaps most important of all, the realization of the power of the axiom had led us to this state of affairs. We are not having as much difficulty picturing constructions using the axiom, and we are continually reminded of what we would have to forfeit if we gave it up.

One of the more recent advances in mathematics occurred when we examined mathematics itself, and recognized that mathematical acitivity was subject to mathematical analysis. Proofs exhibit structure and do not differ from any other mathematical construction. Proofs are abstract structures. When we conceive a proof, or construct a proof, we are using a logic of discovery. The activity is the same as in the construction of any abstract structure. Just as we may prove theorems to describe the aspects of a particular construction we have envisaged, so we can prove metatheorems to describe aspects of proof constructions.

The first question (analogous to completeness) is, "To what extent does a particular derivation capture the proof?" In order to evaluate this, the two objects "derivations" and "proofs," need to be made more precise. We seem to be closer to evaluating the question with regard to logical systems, but what of other areas of mathematics? We have to develop mathematical models of derivations and compare them with mathematical models of proofs. I will return to some of the problems inherent in this task in the next section, but first I will discuss some of the advantages which would result from the comparison.

Formerly, we discussed the notion of construction. We regarded proofs as constructions, although not all constructions are proofs. We will now consider to what extent, or really in what way, does a particular derivation capture a particular construction. One requirement would be that it should lead someone to see a proof. Another would be that it should serve as a reminder for the originator of the proof. When we construct a proof, we idealize, or image, a structure. In a derivation we attempt to capture the structure, or repeat relevant details of the structure. We construct other theorems to describe the structure. We then further analyze the structure in order to complete the derivation.

The emphasis here is on the thought of the mathematician and the way he perceives a proof. The many ways deductions are used to represent derivations also appeal to prejudices of different mathematicians. The degree of formalism in a derivation is a personal choice. This difference of personal choice will be seen to be a central issue separating approaches to the philosophy of mathematics.

The other question (analogous to one of soundness) is, "Is every proof represented by a derivation?" or alternatively, "Is there a formal derivation of every proof?" This question is a decisive one and, as promised earlier, sheds a great deal of light on the disagreements of Hilbert and Brouwer. As we noted, if we restrict the range of mathematics to a finite set of spatio-temporal objects, *viz*., symbols, then there is trivially a derivation for each proof. But if we recognize mathematics as extending beyond symbols, then we have an open question. The response will depend partially on philosophical prejudices we may have; however, if we grant the existence of non-linguistic thought it seems difficult to imagine how this would be captured in a derivation. In the next section of this paper we will examine several options open to us, which have been suggested by Kreisel.

Finite and Infinite -- Limits of Constructibility

The development of a sophisticated "logic of discovery" depends upon a clarification of the concepts of "constructible", "proof", "image", "visualize", and others, which are implicit in mathematical activity. This should lead to solutions for some of the open questions in the previous section. In this section, I will examine some specific attempts to delimit the concept of "constructible."

I will start with some fairly straightforward implications of Hilbert's original thesis.[6] If we restrict mathematics to a study of reconstructed logic, then we encounter some rather paradoxical results. In particular we have to reevaluate our definitions of finite and infinite. This is made clear by analyzing the following question posed by D. Van Dantzig: "Is $10^{10^{10}}$ finite?"[7] If we restrict finite mathematics to spatiotemporal objects, or even to objects in the real world then we are forced to conclude that it is not. Clearly this number is never to be found in the "real world." According to latest estimates, the number of atoms in the universe is $3x10^{72}$ or thereabouts (i.e., smaller than $10^{10^{2}}$).

If we view constructible within the natural numbers as obtainable from zero through the successor relation (the process of adding one), then $10^{10^{10}}$ is not constructible. We will never count to $10^{10^{10}}$, and this is not a limitation of our present abilities. It is in principle impossible to count to $10^{10^{10}}$. No one will ever count to $10^{10^{10}}$. No machine will ever construct $10^{10^{10}}$ out of zero and the successor relation. $10^{10^{10}}$ is, for all intents and purposes, infinite; but it cannot be placed into a one-one correspondence with a subset, and it is therefore a finite number. "It is impossible to construct natural numbers as large as $10^{10^{10}}$ but $10^{10^{10}}$ is a natural number!" (Dantzig, 1955, p. 274). Now we do not view this as contradictory because, as van Dantzig indicates, there is an equivocation on the phrase "natural number." But the problem remains if, as with Hilbert,

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[6] Parts of this section involve technical concepts from set theory, but I have tried to keep this self-contained, and to a minimum.

[7] This is not $\left(10^{10}\right)^{10}$, but rather $10^{(10^{10})}$. This would be 1 followed by 10,000,000,000 zeros.

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we maintain an ultimate reliance on "reality." His strongest argument against the infinite is summarized, "We have now ascertained in two directions, toward the infinitely small and toward the infinitely large, that reality is finite." (Hilbert, 1967, p. 372). But $10^{10^{10}}$ does not occur in reality. Must we conclude that reality is smaller than it, whatever that means?

We have come a long way from 1, 2, 3, infinity. At our level of development, it is not necessary to reconstruct 1, 2, or 3. We do not need to intuit them through the use of insight. It is also clear that we do not need insight for 4, 5, 6, or 7. Our intuitions of all mathematics are dependent on our experience, consequently our intuitions of numbers beyond the initial integers are dependent on our particular experiences. Some people (accountants and good bridge players among others) have little fear of numbers and are comfortable adding long lists and "understanding" many place numbers. 1,000,000 is still pretty familiar to most people in our society. It has the benefit of a single name, "one million." But note as we run out of names, e.g., 1,000,000,000,000,000,000,000,000,000 it becomes less of a number and more of a series of signs with which we operate. Although we may have no direct intuition of the above number, it would be fairly simple procedure to add 12, or substract 4,786 from it. A person dealing with large numbers regularly will have some intuitions about the above number and might compare it to 10^{25} or 10^{30}.

In order to gain a better perspective on this discussion and to lay the groundwork for the discussion in the next section, let us now examine some truly infinite numbers. We arrive at these numbers in the following manner: Consider the natural numbers, 0, 1, 2, 3, 4.... These numbers are presented as a series, a progression. The "..." is used to express the idea that they can be continued indefinitely. These are the ORDINAL numbers, each is formed as an ordered set. We can obtain the successor of an ordinal number by taking the number, i.e., the ordered set, and adding a new element to it, *viz.*, the set containing that number. Thus formed, each ordinal number contains all of its predecessors. ω is the set of natural numbers, and it can be thought of as the largest natural number which therefore contains all of its predecessors.

ω is the first infinite ordinal. We can then continue forming new ordinals by adding a particular way to the collection ω. $\omega + 1$ is a new ordinal ($1 + \omega$ is not). $\omega \cdot 2$ is a new ordinal ($2 \cdot \omega$ is not, $2 \cdot \omega = \omega$). This is a valuable way of comparing infinite sets, but the progression goes on, and on, and on. A first reaction is that the whole mess is absurd. For example, we can talk about the number ε_0 ("epsilon zero"), which is the following:

$$\omega^{\omega^{\omega^{\omega^{\cdot^{\cdot^{\cdot}}}}}}$$

We can describe properties of it, and use it to prove theorems. We can use it to form the following number:

$$\varepsilon_0^{\varepsilon_0^{\varepsilon_0^{\cdot^{\cdot^{\cdot}}}}}$$

There are several relevant questions which we can now ask. To begin with, what is the importance of these numbers for mathematics? This is not an easy question to answer; however, it has been true through this century that investigations into the transfinite have given us a better perspective on the finite. An interesting comment on the above picture is that in a certain important sense we still have ordinal numbers which are the same "size." They are all members of Cantor's second number class. They are all denumerable and they all have the same cardinal number. Mathematicians regard numbers like ε_0 as fairly small, noting that the number of points on a line is greater than ϵ_0

As with the large finite numbers we discussed earlier, dealing with these transfinite numbers has become second nature to many mathematicians. Consequently, the "existence" of larger and larger transfinite numbers depends on the particular branch of mathematics involved or on the intuitions of particular mathematicians. We also have a method, to be discussed later, whereby various sizes of transfinite numbers may be used to characterize basic philosophic positions in the foundations. So, given a positive answer to the first question, namely that transfinite numbers have been essential to the development of our present level of sophistication in mathematics, we have the further question, how far up the transfinite ladder do we at present feel it is safe to go?

Now I will turn to problems implicit in any attempt to isolate the notion of proof, and in any attempt to make mathematical activity a precise object which is itself subject to mathematical investigation. Proofs, as we stated above, are constructions. A prerequisite for an adequate account of proof is an adequate account of constructions.

As pointed out by Kreisel, Bishop, Feferman, Wang, and others, the hierarchies of various versions of type theory present an intuitive foundation for a constructive mathematics.[8] Without going into the details, it will still be possible to analyze some of the results of this representation. Intuitively, we start with a collection of individuals. These are the objects of the world we are looking at. If we restrict ourselves to the world of numbers, a natural choice of individuals would be the natural numbers. We have an important decision as to whether we want to include ω. Russell resolved this by postulating an axiom of infinity, which guaranteed the existence of an infinite number of individuals. At any rate, we will include at type 1 those objects which are primitive, which are not divisible into smaller objects (in this case, they are the natural numbers). We will shortly return to the question of how many there are.

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[8] See especially Myhill and Goodman 1972, Bishop 1967, and Wang 1970.

Type 1 consists of objects which "exist" in some sense of the word. We can then combine these objects in many different ways, forming new sets. The objects of level 1, together with sets defined in terms of these objects, form objects of level 2. We can speak of a totality of order 1 at order 2, because we cannot reach order 2 until order 1 has been constructed! For every level n, we form the next level, n + 1, in the same way. We take all the sets from n and all sets definable in terms of members of n.[9]

But letting this constructive hierarchy extend into the transfinite, we see a direct application of the question posed in the previous section, namely how far are we to extend the hierarchy? The advantage of this cumulative theory is that we have clear lines of demarcation. Corresponding to our own beliefs and needs, we can formulate a particular limit which we deem acceptable.

One of the most significant results for the renewal of interest in the ramified theory is that the intended model of Zermelo-Fraenkel set theory is a cumulative type structure. For example, suppose we accept an axiom of infinity guaranteeing an infinite number of individuals. The ωth level of the hierarchy described satisfies Zermelo's axioms. Without the axiom of infinity level $\omega + \omega$ satisfies Zermelo's axioms. Thus, our decision as to the acceptability of larger and larger transfinite ordinals will give us different set theories.

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[9] This presentation of a type theory represents two major modifications on the Ramsey/Russell simple type theory. (1) There are CUMULATIVE orders. For Russell order n + 1 is made up strictly of sets defined by properties ranging over n. (2) These orders may extend into the transfinite. The impact of this is that order ω includes all sets of the finite orders.

In Hilbert's original program we are to accept only finite reasoning. Strictly speaking this would restrict us from using any general (universal) statements about the natural numbers, and it would therefore eliminate mathematical induction. Although this is a hard pill to swallow, and Hilbert did not stay with it too long, this notion of finite is captured in a simple type structure without transfinite or cumulative types. In order to introduce general statements, Hilbert allowed a restricted form of mathematical induction. This would amount to the admission of level ω, a transfinite type. This would also require a redefinition of finite reasoning, or as Kreisel suggests, to relegating this use of "finite" to the same status of "real," "imaginary" and "complex" when applied to numbers.

One previous way to redefine finite reasoning, oddly enough in terms of our analysis, was to define it as those acts which enable us to visualize a whole structure of concrete configurations. The nature of visualization is not clear. I believe that one important characteristic is that it involves reference to non-linguistic thought which, we have pretty much agreed, goes beyond the finite.

The point of the above discussion is that we can, to a certain extent, use the transfinite ordinals to gain information about theories requiring only smaller ordinals. This gives at the same time both an example of the use of these numbers and a way to characterize some of our assumptions. But it is important to add that this does not cover, as Kreisel intends, the whole scope of our mathematical intuitions. It may allow us to adequately classify those parts of mathematical thought which have been formalized, i.e., those proofs for which we have derivations, but it is not at all clear that it will cover those parts of mathematical thought which are presently beyond this. The most obvious example of this oversight is Kreisel's account of visualizability.

In his discussion of "learning to visualize," Kreisel makes the following reasonable claim, "If one can visualize a structure, then also a sequence of ω copies; more generally if a is a function already introduced, and s a structure, the sequence s, a (s), a (a(s))... can be visualized." (Kreisel, 1965, p. 170). Further, he argues that this leads inexorably to the conclusion that ε_0 is an upper bound on the iteration. (Kreisel, 1967, p.239). It is interesting to note several qualifications which Kreisel appends in both accounts. Consider any structure ξ ("chi"): "For a more realistic theory of actual practice the step from ξ to $\xi \cdot \omega$ may have to be restricted for larger ξ." (Kreisel, 1965, p. 171).

But this is not a case of theoretical versus actual considerations. This is clearer when we examine the second qualification;

> N. B. Mathematicians sometimes speak of visualizing structures which are quite different from those here considered; for instance, in set theoretic arguments, the structure of all denumberable ordinals or of all accessible ordinals. Since their main interest is the validity of their results and not the mathematical activity itself, it does not matter whether they really visualize those structures or only analogous ones, as long as the latter are sufficiently analogous to lead to valid arguments. (Kreisel, 1965, p. 171)

But this is my whole point. This appears to be much more a description of actual mathematical activity than postulating copies of structures. Although ω or ε_0 may be the limits of that one part of mathematical activity, it is a very different kind of activity which we can refer to only by analogy which is at the heart of the non-linguistic thought which is mathematics.

What does it mean for Kreisel to question whether we see a particular configuration or only an analogous one? In practice the visualization of a sequence of copies is typically a start of a procedure of visualization. We then proceed to attempt to abstract from this. We attempt to see common patterns within parts of this structure. Must ε_0 be the limit of this process? Or perhaps more accurately, how good are our intuitions about ε_0 at this time?

In summation several points seem clear. Although we can use a type structure up to specific ordinals to model certain set theories, we still have many open questions. This type hierarchy cannot provide us with a final point of demarcation for our philosophical questions. Philosophy of mathematics does not reduce solely to the question what size of ordinal we are ready to accept.

First, these axiomatizable set theories do not cover our intuitions about mathematics. We can conceive of mathematical objects not definable within these set theories. These objects can be either too "big," or they may be too obscure. Or they may involve the "informal riguor" to which Kreisel alludes.

Second, when we speak about type theories serving as the model (or even natural model) for the various set theories there is a danger of equivocating on the word "model." Although we can have a formal semantic model for the set theory, we may not have a representation of the concepts of one theory in the other. In an extensional language like set theory, we can only show isomorphism, sameness of structure. The intensions of the concepts are lost in the isomorphism. It must be remembered that it is a point of contention as to whether, as Kreisel (I think correctly) points out, the cumulative type structure provides the best intuitive model for set theory. The most important point of this observation is that the decision as to whether it is the best intuitive model clearly does not occur in the language of set theory. Does it occur in any language?

Third, even if the cumulative type structure is successful in the above sense, it is not clear to what extent all of mathematics had been reduced to set theory (a point made in the earlier discussion of the original Logicist program).

Conclusion

The paper was introduced as being programmatic and, although I feel that this was an implicit theme throughout, I want to make the suggested program explicit. I propose a thorough investigation into mathematical epistemology -- as opposed to a study of the nature of mathematical entities. I have presented (1) evidence of historical precedents for this pursuit, and (2) indications of possible avenues of research.

With regard to (1), it is argued that the "schools" in the philosophy of mathematics were originally epistemological in design. They formulated positions on mathematical knowledge and viewed in this manner they are seen to be mutually compatible. With regard to (2), I have focused on the nature of *proof* and its dependence on our notion of *construction*. I propose a broader understanding of the nature of construction. Here it must be understood that "construction" is dynamic -- capable of growth. It can grow in the possibility of new types of constructions; it can grow in the sense that we are not restricted to finite constructions; and it can grow in the sense that a particular construction can change, combine, become more/less precise, or fade into contradictions.

What is needed is a theory of *intensional constructions*. I suspect that this theory will not be easy to arrive at. The most significant progress can only come as a result of combining the four "schools" of the philosophy of mathematics. In the above discussion I only explicitly mentioned the

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Alternative formulations of Intensional Logics have already appeared in print. Especially noteworthy are: Richard Montague "Pragmatic and Intensional Logic" *Synthese* vol 22: 68-94, 1970 and Alonzo Church "A Formulation of the Logic of Sense and Denotation," in *Structure, Method, and Meaning, Essays in Honor of Henry M. Sheffer*, P. Henle, H. Kollen, and S. Longes (eds.) New York, 1951, pp. 3-24, and "Revised Formulation of the Logic of Sense and Denotation" in two parts *Noûs* vol. 7, 1973, pp. 24-33 and vol. 8, 1974, pp. 135-156.

three "brand-name" schools, but the working mathematician did play an important role in the account. At a crucial point it was quite enlightening to refer to Mordell's personal description of mathematical activity.

More to the point, the working mathematician, and the history of mathematics, must serve as guides or check points for our philosophizing. The key to a fuller understanding of mathematics lies in broadening the scope of the philosophy of mathematics to take into account the psychology of mathematical creativity, the logic of discovery, and this is fundamentally an epistemological pursuit.

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Applied Epistemology

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Philosophical investigations sometimes seem divorced from the hard realities of science, which force one to take a position in order to get on with the work at hand. The engineer or physical scientist not engaged in basic research, for example, is almost forced into being a naive realist; one cannot build, design, or analyze an electronic or mechanical device while wondering if the soldering iron, meter, or slide rule is really there.

Once in a while, however, a study that began as a comfortable application of the known to the less well known leads to a philosophical impasse, and one finds himself forced to shift philosophical gears. Perhaps it is a sign of some general paradigm shift that many investigators in many disciplines, even in nuclear physics, are experiencing uncertainty about their working premises, and are beginning to realize that naive realism is not the hard-headed practical man's philosophy it once appeared to be. It may, in fact, create an invitation to illusion.

Other segments in this volume deal in detail with the epistemological position of Piaget, showing clearly that this pioneer has been exploring for years a new concept of knowledge. It may be that Piaget has for several decades suffered an extreme of misunderstanding of his position - or it may be that in his direct approach to the growth of perceptual organization, he has been applying new principles without having organized them into an "official" statement. The subject of this chapter is another approach that has converged to the same general conclusions from a totally different starting point: cybernetics. In this case the epistemological principles have emerged in an explicit form simply as a consequence of following out the logic of a behavioral model.

The model on which I have been working is an offshoot of cybernetics using almost the oldest and least sophisticated of cybernetic concepts: feedback of behavioral outputs to sensory imputs, through the environment. Through a rigorous, and some might say obsessive, application of a simple control-system unit of behavioral organization, I have constructed what seems on first inspection to be a purely hardware model of how behavior works, the kind of model that would make any engineer feel secure.

The C-S Unit of Behavior

The basic unit works like this. A given control system (cs) senses some variable outside itself, the state of that variable being represented inside the system as a perceptual signal. The perceptual computer (the "input function") involved may be very simple, as in a spinal reflex, or it may be complex beyond present understanding, as in a system that can sense abstract variables such as the status of a strategy. In any case, the magnitude of the perceptual signal represents the present-time state of some external variable, quantity, condition, situation, and so on: anything perceivable by a human being. One perceptual signal always represents only one external variable, and is one-dimensional.

Also entering this basic organizational unit is a reference signal (I will say in a moment where it comes from). The reference signal has the same physical form as the perceptual signal, probably a train of neural impulses, and, when fixed at a particular magnitude, represents a particular state of the perceptual signal. Comparison of the perceptual and reference signals (by a "comparator," of course), results in creation of an error signal, a neural signal that continually represents the amount and direction of mismatch between the reference and the perceptual signal.

The error signal - amplified, filtered, and copied into many outgoing channels by the system's "output function" - drives motor behavior.

The most important fact about this model emerges when the feedback path through the external world is introduced. The motor behavior driven by the error signal affects the state of the very external variable that is one of the determinants of the sign[1] and magnitude of the error signal. There is a closed circle of cause and effect.

The net result of putting together these abstract components is a system that behaves in a familiar way: it seeks a goal, the goal being represented by the reference signal. In this model, goals are always goals for perception. We act, if we can, to end up perceiving what we intend beforehand to perceive.

The complete model is made of a whole hierarchy of units like this, a many-levelled organization having many control systems at each level. In general a system at one level makes its perception match the reference signal (or goal perception) it receives from higher systems; its means of doing this is to vary the reference-signals of lower-level systems. Thus behavior is organized into a structure of goals and subgoals pertaining to perceptions of different types. Each class of perception represents a class of external variables (such as color-sensation, configuration, motion, relationship), and the idea that perceptions somehow *correspond* to external variables is the source of the epistemological problem we will explore. Before taking up that problem, however, we have to establish the general picture of this closed loop of cause and effect, which we call a control system.

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[1] Sign" refers to the *direction* of the error: the perceptual signal can indicate an excess over the reference signal, or it can indicate an insufficiency. (Editor's note)

As servomechanism engineers discovered in the early 1940's (and as most psychologists have yet to realize), the consequences of closing this cause-effect circle cannot be predicted correctly by tracing a series of events around (and around and around) the loop. That sort of analysis, when done quantitatively, leads to a model that drives itself, by its own logic, into ever more violent oscillation.

The correct analysis for most cases depends on treating all variables as continuous quantities. It is, after all, only a matter of personal choice whether one calls a variable a discrete fact or a point on a continuum of change, and the control-system model seems to show that the latter is always the more productive - and predictive - choice.

When a continuous-variable analysis is employed, some very peculiar - peculiar yet familiar - properties of this control organization emerge. First, for the behavior of the model to match real behavior, we find that the sign of the feedback must always be negative.[2] A disturbance applied anywhere in the closed loop must result in a reaction by the system that opposes the disturbance.

An unfortunate mistake was made by psychologists who were trying to translate the terminology of cybernetics into something more familiar; in effect, they made a pun on the word positive. Positive feedback is not "good," "supportive," or "firm" feedback: it is error-enhancing feedback rather than error-opposing feedback. Organisms do not react to disturbances with efforts that make the disturbance even more effective; they oppose disturbances. How else could they maintain any recognizable patterns of behavior in a changing environment? Positive-feedback models, on the other hand, do not behave like normal organisms under normal conditions.

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[2] i.e., the value fed back must be different from the value of the reference signal, thus creating a "disturbance." (Editor's note)

Because of the negative feedback, behavior tends to vary in a peculiar way when disturbances occur. The variation in motor activity, however, is not a direct consequence of the disturbance or "stimulus." The physical causal arrow does not run from disturbance to sensory input, from sensory input to motor output, and from motor output to organized behavior. That is the appearance, of course, and it fits the only concept of mechanism known to man before about 1940, but it is only an illusion. The presence of feedback means that the sensory input is the focus of two causal arrows, one from the disturbance and one from the behavioral output itself. The status of the perception involved, inside the system, can be altered either by independent disturbances or by behavior, equally and simultaneously.

The "peculiar way" in which the behavior of a negative feedback control system varies when a disturbance occurs is easy to characterize. No matter how the disturbance (acting alone) might change the perceptual signal, the behavior that accompanies it produces an equal and opposite effect, thus leaving the perceptual signal nearly unchanged. Since all perceptual signals in this model are one-dimensional, all sources of disturbance of a given perception are equivalent, and their effects on perception can be countered by the same error-based output.

An ideal control system will exactly cancel the effects of any disturbance on its perceptual signal, within its capacity to produce opposing output effects. A control system (that is known to be dynamically stable) approaches the ideal as its error sensitivity increases. Error sensitivity is the amount of error needed to cause maximum output; the greater this sensitivity, the less error is required to produce output equal to the magnitude of disturbance, which is the limiting steady-state condition of an ideal system. Thus higher error sensitivity does not imply more output, but smaller error. That is just one of the little surprises in control-system theory.

The biggest surprise, of course, is that control systems control their own perceptions, not their output activities. When an organism reacts to some change in external circumstances, the appearance is that of a direct cause-effect relationship. If organisms are control systems, the reality is that between the external phenomenon and the alteration of behavior is a variable affected by both, a controlled quantity or controlled variable being sensed by the organic control system and being held at some reference level by behavior. Since that quantity does not change (much), it is not very noticeable, but once it is noticed it completely explains why the observed behavior occurred when the external circumstances changed.

As an example, consider this observation. I open a window, and a person sitting by the window gets up and puts on a sweater. The appearance is that the sight and sound of the window being opened triggered off a complex series of responses, in a direct cause-effect way. In control-system terms the relationship is not direct. Opening the window disturbed a controlled quantity, the temperature of the person's skin, and would have disturbed it a lot more if the person had not done something having the opposite effect - put on a sweater. The behavior was protecting a controlled perception against significant disturbance.

Once a controlled perception (or rather, an observable variable postulated to be equivalent to a controlled perception) has been identified, a whole family of cause-effect relationships can be predicted, and explained. Turning up the thermostat may "make" the person take off the sweater; another person might "respond" to the furnace running out of fuel by going to bed. It is practically always necessary to understand the nature of the controlled quantity to make sense of behavior, and people do this even when they don't realize this is what they are doing. Even behaviorists.

Consider walking a dog. Sometimes pulling on the leash seems to stimulate the dog to pull back in a perverse cause-effect sort of way. But the understanding human will realize that the dog's nose is buried in a delicious smell, and the dog is simply keeping that smell nice and strong. Knowing what the dog is really doing, i.e., what he is controlling, one can then predict quantitatively how much the dog will pull against the leash and in what direction. In fact using such carefully applied disturbances is a formal method for determining what a control system is controlling, even a control system that cannot be questioned.

There are many facets to this kind of model of behavior, but we have now established the concepts of interest here. If organisms are indeed organized as control systems, all their behavior is centered around control of their own perceptions, their own inner representations of the external state of affairs. What we observe as overt behavior is merely the means by which organisms maintain this control. Behavioral activities and their effects on the outside world are not the end-product of the nervous system's actions. One cannot really make sense of any behavior until he knows what perception is being controlled by it, and with respect to what reference-level. Only then can one see why a given change in behavior results from applying a given disturbance to the local environment or directly to the organism.

Epistemology and Control

An epistemological problem arose when I tried to do something with this model that is not normally done with behavioral models; i.e., when I tried to apply it to all organisms, including the experimenter as well as his subject. If it is a valid model of organisms, one can hardly claim exemption from it and still defend its generality.

The problem arises from asking how an experimenter could establish the fact that his subject is behaving as a control system with respect to some observable variable. The obvious procedure is direct, in that its object is to see whether behavior satisfies the definition of control behavior, control of input through opposition of disturbances of that input.

Unfortunately, one cannot tell by inspection which aspect of the environment or organism-environment relationship is in fact being perceived at the moment by the organism, much less whether it is being controlled. The mere fact that a variable remains constant proves nothing; it might do so with no organism present, or its constancy might be coincidental to the control of some other variable. When starting from scratch, the experimenter has no choice but to hypothesize a controlled variable, and then try to test the hypothesis.

The test, fortunately, is fairly simple in principle, although applying it requires some inventiveness. Once a potential controlled quantity has been selected, one simply applies disturbances directly to it (for instance by varying other quantities contributing to its state) and sees whether or not the organism's behavior opposes or cancels the effects of the disturbance. If the definition is not quite right, there will be ways of disturbing the proposed controlled quantity that succeed just as if the organism were not there. In that case, one alters the definition to exclude dependence on that type of disturbance, and tries again. The procedure is finished when every possible way of disturbing the variable is met by equal and opposite effects solely due to the organism's actions.

Of course one's final definition may include more than one variable under control by the organism; one may be mistakenly unifying variables that are separately controlled by the organism. The final definition might be a transformation of the actual controlled variable, or it might leave our perceptual functions common to observer and subject. Presumably, experience will show up some of these errors sooner or later. One wonders, though: can this test ever reveal them all?

But that is not how epistemology comes in - that would be too obvious and anticlimatic. The subject of epistemology comes in when we recognize the experimenter's actions as another example of control behavior.

In that segment of experience which the experimenter calls his "mental" world, he is aware of a goal-principle, principles being one of the higher-level classes of perception in the nine-level model (Powers, 1973). The principle or generalization he wants to perceive could be stated, "Disturbances of controlled quantities create equal and opposite effects from behavior." Once he had grasped this principle, he would not describe it; it would become a unitary perception, non-verbal.

What he does perceive at this level would at first have to be stated as a different principle: the disturbance he is trying out is only sometimes opposed, or never. By altering his choices of goals for lower-level perceptions - strategies, relationships, events - he finally makes the perceived principle identical to the goal principle, eliminating the error.

What we mean by mastering an experimental technique is learning how to perceive a principle, and how to select lower-order goals in such a way as to diminish the error. Learning is involved in establishing this control organization in working order, but once the experimenter "gets the hang of it," he is not learning any more but simply behaving as a complex control system, controlling the multilevelled perception called, in this case, "the test for the controlled quantity."

When the test procedure has been carried to a successful end, the experimenter will find that his own perception of the controlled quantity is stabilized against disturbances by the subject's behavior. That is the point where he feels that he now "knows what the subject is doing." He has found an effect of the subject's motor activities that remains the same despite drastic and independent changes in the environment situation.

This, of course, is what any observer is doing when he recognizes a "behavior pattern." A study of behavior has never been a study of outputs. Outputs do not repeat; only their consequences do, and those consequences are defined by what the behaving system is perceiving and controlling. The observer's perceptions do not define the behavior of the subject; the subject's do.

We now have a situation in which at least one variable in the environment, the controlled variable, is represented as a perceptual signal in each of two brains, the experimenter's and the subject's. The processes by which the perceptual signals are derived from sensory contact with the environment are at least similar, the more so if the perceptual signals are invariant with respect to the spatial orientation of the observer. Under a large variety of circumstances, these two perceptual signals will covary, or conversely, be stabilized.

Now the question we have been approaching. Of what are these two perceptual signals a function? The naive realist answers, "Why, the controlled variable, of course." But this test does not reveal any controlled variable; it reveals two covariant perceptual signals, but not the thing of which they are both presumably - a big word - a function. We can call in any number of colleagues, teach them the test, and get them to establish their own perceptions of the controlled variable, but numbers make no difference, concensus does not help. We still have no way of knowing what is actually being controlled in that hypothetical reality we think of as existing independently of our experience of it, the reality between your experience and mine. We are all experiencing an analogue of that world, in the form of signals in our brains: the atoms of the world of experience are neural signals. The model tells us that both the model and the experiences we are modelling exist inside our brains (it does not tell us if we should believe this).

Even cross-correlations among senses, or the interposition of artificial sensors, does not enable us to cross the barrier. True, we can cause our visual perceptions to change (by exerting muscular efforts) in ways that lead to changes in tactile perceptions: that may reassure us that what we are seeing is "tangible" (which means, perceivable through the tactile sense), but only if we are desperate for reassurance that we experience Reality Itself.

All right, one may say, it may be true that we cannot see through our perceptual transformations to the cause of perceptions. But does not the fact that human beings share such similar classes of perception show that our perceptual analogues are very much like the real world? I thought so, once. But I ceased to think so when I realized that human beings can control perceptions, through very concrete behavior, that cannot possibly have external counterparts. Find one example and you have a thousand.

As everyone knows (all accusations of prejudice are rejected in advance), home-made soup is always too salty. Different soups need careful adjustment of salt-content, so as to create just the right overall taste. This taste is easy to recognize and, by means of a salt-shaker, to control, although unaccountably hard to communicate to the cook. What is being controlled apparently has no objective significance except to another properly organized nervous system. I presume that example was on familiar ground; but one can go further. With practice, one could learn quite skillful control of the flenishness of his environment. Flenishness is defined as the sum of a Fahrenheit thermometer reading and the time indicated on a clock in hours and fractions of hours. Since these are both meter readings, they must be objective measures of something.

Suppose one determines to keep his environment's flenishness at a reference-level of 72. This can be accomplished in a number of alternative ways (a fact that helps assure us of the reality of flenishness). One could, for example, manually adjust the clock reading, while watching the thermometer as well, keeping the sum at 72 as the thermometer reading changes. One could, if this seems like fudging, carry the thermometer about while watching the clock. Toward noon it might be necessary to put the thermometer in the refrigerator to keep a flenishness of 72; the distribution of flenishness is not, apparently, uniform in space or time, but then what is? As a third alternative, one could leave the thermometer and clock alone, and control flenishness by manipulating the air-conditioner and furnace with dedicated disregard of comfort.

Exploring the absurd can sharpen one's eye for it. One does not need to invent outlandish controlled quantities in order to find enough examples of perceptions that can be controlled but have no conceivable reality outside the human brain. Consider an architect designing a new house. He wishes to make the drawing attractive: how shall he adjust the attractiveness of the drawing? By changing proportions, by adding shading, by drawing in a few trees, until the attractiveness is at the architect's reference level for it. The client, of course, will prove boorishly insensitive to the obvious attractiveness of the drawing. Is the problem a lack of sufficient attactiveness? Not at all. It is the absence of an external referent for the term.

Our language offers a multitude of easy ways to speak of our perceptions as if they were really where they appear to be - Outside. We can speak of provocative behavior, pleasant surroundings, understandable explanations, valuable books, gigantic people, strong tastes, scintillating conversation, irritating habits, and even "stimulus value." The list is endless, for it reflects the naive realism which was all our ancestors were capable of. The list "reflects" the habits into which even the most dedicated study of epistemology has fallen, habits that would be exceedingly hard to change. It is frustrating to realize that one's very thoughts flow onto paper in a language that asserts what one knows is not the case.

We 20th century brain-modellers-cum-psychologists must, it seems, resign ourselves to a split-brain sort of existence, hoping that language will evolve enough, in time, to save our descendants this incessant feeling of hypocrisy. In the meantime, we can always talk in algebra.

If there is any validity in this control-system approach, we must face up to a very serious charge concerning human experience. While the model does assume that perceptions are functions of something Outside, it offers no assurance that there is a specific external counterpart of any human perception, from elementary sensations on up. It is quite consistent with the model to suppose that human perceptions are arbitrary functions of external variables. These functions become organized because of their survival value (here we are not in disagreement with B. F. Skinner), but the criteria defining "survival value" are within the organism, themselves the results of a control activity in which an entire species participates.

One central concern of this volume, learning, has only been mentioned in passing. This primarily is because the epistemological problems we have been looking at are problems that confront us in our roles as examples of a hierarchical system organized about perception. The control hierarchy we have been looking at has fixed properties; I deliberately left out of this performance model any capacity for self-reorganization, partly just to see how far one could go in accounting for behavior without introducing the capacity for change.

After postponing the question of learning until the performance model has been built, one finds that the question has changed. As pointed out in an earlier example, "learning" what another organism is controlling can be reduced to an algorithm, a procedure that can be learned. Surely learning the algorithm and applying it are two different processes! Once we understand just how much of behavior can be brought under control-hierarchy paradigm, we begin to see that a very large part of what is called learning has two aspects to it: that which is learned, and the process by which it is learned.

What can be learned? Practically everything. The products of the learning process include rationality, strategies, data about the outside world, modes of interpretation, principles, and just about everything else we consciously experience. What cannot be accounted for by the application of these modes of thought and action? Learning itself. The process that brings these organizations into being out of chaos.

Control theory can help us define the goals of learning in an orderly way, and may be able to help us organize what is to be learned into a sequence that fits the sequence in which the hierarchy must be built up (you can't teach principles first - not if you want to teach what you intended to teach). But at least as important, it shows us what is being left out of the teaching process. Even without control theory, we know how to define what is to be learned - understanding, or perhaps performance objectives. We know how to set up a situation that requires learning to happen. But we do not know how to initiate, direct, or sustain that process itself. It happens, or it doesn't happen. Reorganization commences, or it does not commence. No teacher was ever told how to make it start.

This process of change is not an epistemological problem. It is not concerned with the product of learning, which is the perceptual hierarchy that does or does not represent a world outside of perception, and which is part of a control-system organization. Because of that fact, an approach to the subject of reorganization, and especially to its relationship to human needs and built-in human properties even deeper than what we call needs, may offer a totally new approach to epistemology. Perhaps the question we ought to be asking is: by what criteria does an organism decide that one representation of reality is "better" than another? We can easily say that a taste tastes "good." We cannot so easily ask whether "good" is good. This brings us close to questions about our very nature, and its relationship to all living things.

Maybe this is the most suggestive result of my epistemological investigations: recognition of the fact that the process of change is not part of the epistemological problem, but creates the kind of organization that puzzles about epistemology.

Radical Constructivism in Psychological Explanation: A Synthesis of Negative Feedback and Operationism

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Some interesting implications for explanatory systems in psychology emerge when three modern concepts are integrated: radical constructivism, an epistemological doctrine orginating contemporaneously in physics and psychology (Bridgman, 1927; Dewey, 1929; Piaget, 1936)[1] but only recently making an impact on theoretical psychology (Powers, 1973; von Glasersfeld, 1974a); negative feedback, an information processing sequence first formalized by N. Wiener (1948); and operationism, a pragmaticist interpretation of the nature of scientific concepts (Bridgman, 1927).

The radical constructivist philosophy serves, in this context, as an epistemological framework for a discussion of the role of feedback in theories of behavior and for the application of operational analyses to cognition. It posits that biological organisms build their "realities" or "environments" and respond to those constructions. The enviroment, then, is a model which is internally organized and which cannot be directly known. The philosophical origins of this position lie in Bishop Berkeley's rationalist dissociation of sensible object and material object (which is unknowable). As an epistemology, radical constructivism cannot be defended with "evidence" derived from empirical findings since evidence is itself the product of the constructor. Rather, the test of constructivism lies in the generation of a system of formal descriptions of the psychological processes by which the organism constructs a world. As will be shown, these formal descriptions are subject to tests.

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[1] While it is possible to interpret the positions of these authors as constructivist (i.e., interactionist), rather than as radical constructivist, the latter interpretation is made here. The former interpretation is that there exists a structured environment which the organism restructures through cognitive acts.

How can a system be built which can account for human experience without describing properties of an environment? Immanuel Kant's description of an interaction between "the understanding" and "intuition" (or sensation) approached the constructivist problem in this way:

> All our judgments are at first merely judgments of perception; they hold good for us (that is, for our subject) and we do not till afterward give them a new reference (to an object) and desire that they shall always hold good for us and in the same way for everybody else; for when a judgment agrees with an object, all judgments concerning the same object must likewise agree among themselves, and thus the objective validity of the judgment of experience signifies nothing else than its necessary universal validity. (Section 17, Second Part of the Main Transcendental Problem, Prolegomena).

His proposed correlation between *a priori* mental categories and independent objects, however, reflected a hesitation to make the full constructivist leap. It is suggested here that a constructing system can be described as one which monitors its own processes via negative feedback. In a mechanism incorporating negative feedback, a sensed discrepancy between a reference condition and the actual condition of a variable results in activities aimed at reducing the discrepancy. "Actual conditions" refer only to conditions of variables which a sensor can discriminate, and not to a property of an environment. Far from being a philosophical position, negative feedback is a means of describing control loops in mechanical and symbol-manipulating systems ranging from the Egyptian waterclock to the "DO" loops of computer programs and to man's central nervous system (Mayr, 1970). Powers (1973) notes that when negative feedback loops are arranged in a hierarchical manner, rather complex processes occur.

The creation of novel activities by which discrepancies within a feedback loop are reduced, is the result of a synthetic operational process. In its inception operationism was conceived as a technique by which scientific terms were defined in reference to the physical activities employed in determining their values. "Length," so defined, becomes a concept with no connotations beyond the acts of measuring. In two papers, W. R. Garner (1954; Garner, Hake Eriksen, 1956) has noted that the plural reference to act_s_ is crucial to an operational definition of a concept. Citing an often-quoted phrase of Bridgman, "The concept is synonymous with the corresponding set of operations" (1927, p.5), Garner suggests that one of Bridgman's intents was to delineate the implications of what Garner calls "converging operations": two or more actions (operations) are necessary to verify alternative hypotheses and define a scientific concept. The resulting concept is not synonymous with _an_ activity. Length, when characterized by both direct measurement and triangulation, becomes a concept which represents what these operations have in common -- an outcome. The operations, then, are said to "converge" on a single hypothesis, and the outcome becomes a scientific concept. Though Garner refers only to methods of science, his notion of converging operations may shed light on the construction of knowledge in the organism. In fact, if one closely reads Kant's statement (above), the germ of converging operations, in multiple "judgments," is apparent.

Radical constructivism and negative feedback are linked by a fundamental notion. Both describe systems which are closed. Radical constructivism is a closed-ended epistemology -- dualist accounts of knowledge are rejected. The nature of the experienced world must be explained by describing only the experiences. The feedback loop is closed in the sense that control or causal flow is circular. If state X at time t increases the probability associated with the occurrence or inhibition of state Y at time t + 1, state Y recursively acts upon state X at time t + 2 either directly or indirectly. This can be contrasted with an open unidirectional flow in which state X_t acts upon state Y_{t+1} which in turn acts upon state Z_{t+2}. While in the latter system, X can be said to cause Y which in turn causes Z, an absolute distinction between cause and effect within a closed-loop system is inappropriate. The construct of an open system has been used as a context for theorizing about psychological organization (von Bertalanffy, 1967, 1968). According to this view, the organism does not seek equilibria, and is not composed of reversible processes. New energy is envisioned as constantly being reorganized by the biological system as it flows through. But while physiological systems might be described as open, the difficulty of defining the source and forms of energy input in "open" psychological systems makes the closed loop an attractive alternative. In the closed-loop system, a reference condition and an actual condition are generated outside of a single loop. As will be shown later a radical constructivist system must be composed of an hierarchical arrangement of loops. In most cases, any one loop will have its reference and actual conditions determined by other loops. Ultimately, however, the system does have two ends: a loop (or set of loops) at the top of the hierarchy which "receives" a reference signal from an unknowable source, and a loop (or set) at the base of the hierarchy which "receives" a signal representing an actual condition, the source of which is also unknowable. In a sense, then, this closed system has two ends, but since no "transmitters" can be specified which send the signals "received" by the hierarchy, discussion of a "flow" of energy through the system is not enlightening. In fact, the term "receives" is inappropriate, since the transmitter can not be identified. Unfortunately, English, designed by realists, leans heavily on dualistic relations in which every effect implies a cause. The "receiver," as used here, must be thought of as operating at a boundary with only a single side. While closed feedback-loops individually seek a static equilibrium, in combination they can be depicted as irreversibly developing (i.e., reorganizing) in active searches for equilibria. A dynamic state is maintained not because stimuli are constantly changing (stimuli have no meaning in a closed system), but because of the nature of the system itself. While this is not a causal explanation of system dynamics, no specification of cause is necessary. We can say that the feedback loops exist in relation to one another such that no one equilibrium condition satisfies all loop goals, leaving the organism performing a constant "juggling act."

In suggesting that radical constructivism provides a context for describing behavior, a rejection of dualism in favor of a description of mind demands that a model for mental processes be found. Such a model would retain radical constructivism as a viable epistemology underlying the postulation and testing of scientific theories. Since negative feedback can be described as a closed loop which needs no qualitatively specifiable input external to the loop, it is an appropriate candidate.

When one conceives of the human organism as a closed loop, the intuitive (and, within psychology, traditional) distinction between self and environment is destroyed. What the empiricist views as environmental stimuli, the radical constructivist (hereafter simply referred to as "constructivist") views as organizations which both depend upon and influence the organisms' consequent actions. According to a constructivist position, the organism no longer is characterized as reacting to influences outside, but itself determines which events are perceived and which actions are produced. For example, imagine this situation: a car heads towards you; you step out of its path. Observing the sequence, the realist describes the interaction: "The car made him move." The constructivist would describe an internal event: "He had a perception of a car approaching him. Knowing that he might be injured, he stepped out of its path in order to alter his perception."[2] What caused the perception of the moving car? First, there is an "unknown."[3] Second, a cognitive structure created a perception. There can be no qualitative nature attributed to the "unknown" apart from the organism's perceptions. Since the "unknown(s)" are operationally indistinguishable from perceptions, references to them as causes serve no function. They have no explanatory potency. Analysis of the organism as composed of closed loops allows for the positing of "unknowns" outside the loops. The meaning they are given, however, takes place within the loops. From the psychological standpoint, the environment is reduced to a conceptual model. It loses its objective nature, since the only way of sensing it is by an act of the organism. The constructivist position implies precisely this loss of objective reality external to the self.

Constructivism and Psychology

Constructivism provides more than a palatable philosophy. As Bridgman recognized, it provides a firm foundation for scientific inquiry, including psychological investigation. Experimental psychology began with an attempt to measure subjective experience (Gustav Fechner's psychophysics); to make public the private mind and to make experience itself a field subject to rational investigation. The problems inherent in hypotheses of unobservable mediating mechanisms which allow the prediction of observable measures are amplified in psychology. The "mechanisms" are descriptions of self. The process of observation of responses does itself require the functioning of those mechanisms. The psychologist is easily intimidated by the circular relation between observer and observed since "discovery" of "external reality" in a closed loop is impossible. When science is aimed at characterizing a Newtonian world consisting of physical laws external to the human "lawmaker," subjectivity of observation becomes problematic. Psychologists' responses to this problem have led to two epistemologically tenuous poles: (1) a retreat to the realism of the behaviorist who attempts to define minimal units of observed organismic responses correlated with environmental stimuli, who minimizes potential difficulties involved in defining observations as if they were objectively independent of the observer, and who ignores the mentalist implication of terms like "correlation," and (2) a rejection of formalizable explanatory systems in retreats to both rational introspective analyses and irrational subjective understandings (e.g., Titchner's "structuralism" and Weber's "verstehen"). These latter approaches, of course, leave the theorist without recourse to experiment design or operationally testable hypotheses.

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[2] His moving, in turn, strengthened his original concept of car, since he saw it pass and, as expected, was not harmed. If he had moved, watched the car pass, and still been hit, he would have altered his concept of "car" (and perhaps of "object").

[3] Ernst von Glasersfeld notes that to say an "unknown" occurred or that the unknown is an event would suggest that the "unknown" has coordinates in space and time. However, space and time, like all human notions, are not attributes of an external world. The constructivist simply accepts that a change has occurred without a cause.

Constructivism avoids problems incurred in dealing with the predicament of recursive references in an objectively "real" world. By admitting that the environment can not be separated from the organism which organizes its perceptions of it, the constructivist faces the problem, not of describing the relation between an organism's responses and external stimuli with specifiable causal influences, but of describing the organization of the perceiving system. In a universe of unknowable environmental properties, an autonomous organism determines the arrangement of sensed properties and responds to its own arrangement. Psychologists, however, are able to describe their world and their "selves" since, having admitted that there is no objectively knowable world or self, the goal of explanation becomes prediction of perceived events from internally consistent theories, rather than prediction of absolute events from absolute principles and specific facts. In the same manner, understanding in the organism becomes a search for coherence, or equilibrium between expectations and actualities.

The implication here is that both the organism under study and the investigator function as hierarchical systems. An "arrangement" of sensed properties (the states of lower-order loops) may serve as the "actual condition" for higher-order feedback loops. Undesirable actual conditions (from the point of view of the higher-order reference levels) are modified. Upon returning home after work, Mr. Jones may decide to open his front door (a previously constructed reference condition). At level 1, he compares the actual condition of his front door to his reference condition, "See an open door." A comparison which results in a mismatch at this level generates a reference condition for a lower-order loop at level 2, perhaps one which can be stated as "Have your key." This reference condition is compared with another actual condition (does he have the key?). If there is a match at level 2 (he has it), the door becomes open and a new actual condition at level 1 has produced a match. The higher-order loop generated a new actual condition for itself by using a lower order loop. Without a proposal of hierarchies of feedback loops, which do the constructing, constructivism would appear to be a mystical conception inapplicable to any branch of science.

Constructivism and a Developing Organism

The following example demonstrates the application of a constructivist approach to analysis of behavior. Not knowing an objective universe, the developing organism constructs one in a manner consistent with Garner's notion of hypothesis-testing via converging operations. In the neonate, internal organizations or "cognitive structures" (and resulting expectancies) arise when two or more converging physical activities determine a single satisfying result.[4] Jean Piaget, an epistemologist interested in attacking the problem of the structure of knowledge by examining human ontogenesis, proposes that it is precisely the coordination of actions which leads to the discovery of single results, or "invariants." For example, the infant begins life with simple reflexes and nonintentional muscular movements. Eventually, it constructs invariant action-patterns, shapes, objects, temporal sequences, etc., by coordinating these physical movements. Piaget suggests that the processes of coordinating actions become "interiorized" as higher-level processes which coordinate mental concepts through a synthetic process called "equilibration." In the infant, the reflex actions of sucking and random movements of the head can become integrated in a single functional mental unit or "schema" for "searching" since combined they frequently produce milk-sensing (Piaget, 1963). The physical operations of sucking and moving converge and a novel structure is formed. The behaviorist might describe the repeated application of a discrepancy-reducing schema simply as a sinuous statement of reinforcement effects on behavior. But what is the reinforcement? As Powers suggests, it is an internal registering of the reduction of difference between two perceptions -- one of perceived actual condition and another of reference, or desired condition. It is not the "N" ounces of warm milk which the observer perceives.

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[4] "Satisfying" can be read as either "need reducing" in the Hullian sense, or as "equilibrating" in the Piagetian sense, the latter referring to the immediate drive value of the reduction of cognitive uncertainties.

When Garner used the term "converging operations," he meant the following: "...any two or more experimental operations which allow the selection or elimination of alternative hypotheses or concepts which could explain an experimental result" (Garner, Hake, and Eriksen, 1956, p. 151). For the infant, the result (milk-sensing) which occurs is accidental initially. The hypotheses which are formed are primitive and biological: repetition of the activities preceding the satisfying result will increase the probability of its reoccurrence (Thorndike's law of effect). The activities converge on the hypothesis (suck + move results in milk-sensing) and a action-concept is created. If milk-sensing is characterized as a goal, the infant's creation of search coordinations yields a mechanism which reduces the discrepancy between the goal and an actual condition of food deprivation. The constructed coordinated actions result in a pattern which delineates a negative feedback loop. When milk-sensing is occurring, the actual condition matches the reference condition, or goal, and the "search" motor schema is inactive. Or when the reference level of milk-sensing is low (i.e., the infant is no longer hungry), a search is not required. But when the actual condition does not match the reference condition, the schema goes into play in order to produce a match.

The three concepts defined in the introduction enter in this description of the infant's mastery of a world. The organism constructs its environment. It does so by creating its own converging operations (mental or physical) which lead to satisfying results. These converging operations are actions, percepts, or concepts which serve as mechanisms by which discrepancies between goals and actual conditions in a feedback loop are reduced. The converging operations called "search" reduce hunger in the infant, for example.

The search for operations which do converge itself describes a feedback loop in which the actual products of combinations of operations are compared with the predicted/desired reference state. Combinations which are successful (i.e., which converge) are synthetically retained and repeated. Negative feedback, then, is required to describe both the application of products of converging operations and the means by which the converging operations are constructed; that is, the application of knowledge and the acquisition of knowledge. The intrinsic nature of the constructive process is developmental since feedback as a model for application and discovery of converging operations applies to the organism at all points in development.

While the example of the actions of a nursing child serves to illustrate the nature of the construction of invariant operational combinations via the equilibrations within feedback loops, operations in the organism need not be physical actions. Piaget (1970) defines four fundamental characteristics of the mental operation: (1) "an operation is an action that can be interalized," (2) "it's a reversible action," (3) "it always supposes some conservation, some invariant," (4) "no operation exists alone" (1970, pp. 21-22). While these functions coincide with the present description of converging operations, they add the feature of internalization. The products of physically converging actions can become the elements from which new nonphysical convergences are constructed. Again, the notion of hierarchy becomes integral in interpreting constructivism. Here it is applied to the construction of novel feedback loops rather than only to the use of pre-existing loops. This additional feature, internalization, provides for the construction of structures made up of hierarchical arrays of operations and of the feedback loops which they compose. The physical "search" schema is constructed of converging operations. It becomes, in addition, a single operation which in combination with other operations converges upon a new result. "Search" in combination with "grasp" may yield new schemata -- a primitive construct of object, for example. This is not to say that this particular combination occurs. But it provides a theoretically fruitful conception of the means by which a reality is constituted.

Constructivism and Solipsism

As noted, feedback involves the determination of a discrepancy or equivalence between a reference condition and an actual condition. Since the "actual condition" is itself constructed (e.g., it may be the state of another feedback loop), the notion of feedback when applied to the organism-as-self is potentially solipsistic. One might assume that there is a real world existing beyond the action of the perceiver, the knowledge of which is forever denied him. But when feedback is coupled with the postulation of objectively undefinable "unknowns" external to the organism (that is, there is no "real" world to be known), this elaboration of constructivist epistemology does not terminate in unproductive solipsism. At the level of sensation, an actual condition can be distinguished from a reference condition since only the actual condition originates from the condition of a lower-order loop. At what is theoretically the lowest order loop, the actual condition originates with the undefinable unknown external to the loop. Imagination, memory, and concepts can be distinguished from sensory processes and a circular reasoning dilemma is avoided. Elementary "actual conditions" are sensations. For the realist, sensations reflect the world. For the constructivist, the sensations co-occur with unspecifiable unknowns. But these unknowns are not the world. The world is the product of combinations of operations -- and can be no more than this. The "self" is not reduced to an amorphous consciousness or a Cartesian mind in isolation. Similarly science (and particularly psychology) remains a viable means of inquiry.

The organism operates on its sensations (which co-occur with unspecifiable unknowns), organizing them through comparisons among mental and physical actions, and constructs what appears to be an ordered world external to its self. This view of a real world outside one's own "self" is quite useful in most cases. Having evolved expectancies based on past constructs, the experiencing man finds that his actions (including concepts and perceptions) yield matches with his goals. But confronted with what he later calls an illusion or a misunderstanding, he discovers that he had indeed constructed that part of his world. The construct only becomes apparent in the rare instance when two operations (e.g., seeing and touching) do not converge upon an expected single result. Usually once a concept is formed, we do not retest it by comparing actual outcomes of converging operations with predicted outcomes. There is no opportunity for feedback, and the illusion is not discovered. For example, you do not remain in the path of the car to test the tactual aspect of your concept operationally. Instead you watch it move, using a visual operation (seeing it approach and pass) to verify the tactual expectancies attached to a concept of car. The operation which actually does test the tactual expectancy (i.e., touch) is not applied, however. A match between the actual event and your stored concept of car is inferred (or more specifically, induced), not verified, since operations which converge are not used.

Precisely in the manner that the individual constructs his invariants (e.g., concept of car) through his acts upon them (e.g., perceptions such as touch, vision) and has only those acts to define them, the scientist (ideally) constructs his theoretical invariants and defines them only by the acts of deriving them. How does he define the acts themselves? A. C. Benjamin (1955) addressed the question in describing operationism, pointing out that a longstanding conflict within empiricism remains in Bridgman's work. While John Locke, for example, postulated relative "complex ideas" derived from the constructive actions of man, they were formed from "simple ideas" that were absolute passive mappings of the world. Bridgman adopted the notion of "complex ideas" derived from operations relative to each other, but the operations themselves were not derived from any absolute sensations. How, then, does the scientist (or organism) determine which sensations or experiences to accept as physical operations and which to reject as mental constructs? If we "make" experience, how do we determine the foundations of operations? Bridgman suggested that operations must ultimately be defined nonverbally in physical actions which can be imitated by others. For Bridgman, when he discussed scientific methodology, public consensus lay at the root of operationism. The "other man," then, provides evidence against which the scientists' ideas are compared. Reference conditions such as "replicability" and "alpha-level" are compared with actual conditions and the validity of scientific concepts is tested. The assumption the scientist makes is that other men are more real (i.e., are lower-order constructs) than are his scientific constructs, since the men are used as evidence to test those constructs.

While the scientist can obtain or assume public consensus for definitions of methods and concepts, the theoretical organism, given no _a priori_ environment containing a public, can not. Both the individual as a scientist constructing a theory and the individual as an organism constructing an environment must reject notions of _real_ worlds beyond understanding. What the organism must fundamentally assume is that his own physical actions and the perceptions which guide those actions co-occur with eternal unknowns external to himself. While this assumption is also philosophically necessary for the scientist, in order to *practice* science he must also assume pre-verbal minimal actions, or operations, and a public as a lower-order concept which distinguishes idiosyncratic constructs from replicable actions.

Criticisms of Feedback as a Constructive Model

W. R. Ashby (1952), Miller, Galanter, and Pribram (1960), and most recently, W. T. Powers (1973) make eloquent cases for the applicability of the negative feedback circuit as an explicit unit by which behavior can be empirically analyzed and described. In other models of cognition these units are commonly acknowledged through references to "control programs" or "coordinating programs" or are simply implied by the assumption of goals. But feedback as an explanatory model has been criticized. Since the notion provides the lever by which the constructivist epistemology can be applied to behavioral analyses, its validity is critical.

There are two common criticisms of feedback units as explanatory principles of human behavior: (1) the criticism, invoked by advocates of a dualist mind-body separatism or of an antidualist mind-as-body synthesis, that mechanical or biological feedback either can not apply to analyses of mind or supplants analyses of mind; (2) the criticism that feedback modelers are left with the problem of describing the origin and nature of "goals" or "purposes." This latter issue results in statements that "A scientific analysis of behavior dispossesses autonomous man and turns the control he has been said to exert over to the environment"(Skinner, 1971, p. 196), or that feedback models potentially describe the elasticity of rubberbands as well as of human behavior and are thus inadequate for describing the quality of human purpose (a position adopted by Richard Taylor, 1950).

The Mind-Body Problem

Several philosophical arguments suggest that while statements about (or descriptions of) mind and brain are not identical, the consciousness (e.g., experience, sensation, mental imagery) is potentially caused by brain processes (Place, 1956; Putnam, 1969; Smart, 1962). Others (e.g., Feyerabend, 1969) suggest, in addition, that mind (and psychology) is not a useful construct since the material universe can be described only with the assumption "...that the only entities existing in the world are atoms, aggregates of atoms, and that the only properties and relations are the properties of, and relations between such aggregates"(Feyerabend, 1969, p. 82). The former case allows that mind may be described by feedback because brain processes may be described in this manner. The latter suggests that neural (material) feedback circuits (rather than psychological circuits) are the only valid constructs. This second position is defended on the ground that references to mental entities as minimal units of theory construction lead to a constructivist dilemma of circular reasoning since physical entities are reduced to the products of these mental entities and can not be verified "objectively."

Contrasted with the mind-as-body (or body-causes-mind) position, the constructivist position maintains that mind-causes-body. Brain processes result from experience since it is the mind which has constructed the concept of a brain with processes. This position, of course, is antithetical to the materialist concept of man in the physical universe, for it is the physical universe which is modeled or constructed in man. As noted earlier, the notion of negative feedback gives the constructivist the ability to build and verify concepts by comparing expected outcomes with actual outcomes. "Feedback" is a construct of mind which is imposed on experiences --

including experiences which yield a construction of "brain." Whether the imposition is a successful one can be operationally tested, since predicted novel experiences derived from converging operations can be compared with actual experiences.

Mind-as-body proponents suggest that explanatory terms must have physical referents or that mental terms must correlate with physical ones. The constructivist also deals with this problem of elucidating explanation. Given that terms referring to both the physical and mental world are themselves mental constructs, which terms are most appropriate for describing the mind? Nagel's conception of "connecting principles" (1961) which serve to correlate concepts (e.g., mental and physical) suggests that the question need not be answered. If both methods of describing the world lead to theories with predictive power, they will eventually lead to the discovery of meta-principles allowing for transposition of statements from one area to another. The constructivist would agree with this view, not because accurate descriptions of a Laplacian deterministic world must be congruent with one another, but because the describer devises concepts which, by virtue of arising from a single biological or psychological system, share identical logical properties. The suggestion of this paper, of course, is that two of the fundamental connecting principles may be negative feedback, as a description of a particular type of circular causal flow, and converging operations, as a description of the means by which actual states become more similar to reference states, in both psychological and biological (and perhaps chemical) systems, and in mechanical control devices patterned after those systems.

Purposiveness

The second criticism -- that feedback does not account for the origin or nature of purpose -- must also be considered if constructivism can serve to account for the nature of thought. The behaviorist view proposes as a premise that man, like the rest of his universe, is not purposive, but only appears to be because the method by which antecedents of behavior influence behavioral consequences has not been clarified. Because man is not purposive, that is, his actions are not caused by expected future results, his behavior can be explained by unidirectional causal mechanisms. On the other hand, the phenomenalist argues that man is future-driven and thus is not mechanistically identifiable. The "rules" or principles necessary to identify specific physical mechanisms would not apply to man (Dreyfus, 1972, recently made an ambitious defense of this view). This conflict between the subjective experiences of will (and choice) and the knowledge of non-biological entities as unconscious rule-guided mechanisms has led to constructivist reinterpretations of teleological cause. Monod (1972) has proposed a reinstatement of the term "teleonomy" for descriptions of individual behaviors and biological evolutions which appear to be future-driven (in the manner of Aristotle's "final cause") but which in fact are past-determined. In the behaving organism, final cause appears with the representation of future-expectancies in an organism which acts to instrumentally fulfill them (von Glasersfeld, 1974).

Information processing models in psychology all appear to circumvent the disconcerting issue of the origins of the "future-expectancies" by proposing plans of some sort (usually a hierarchically dominant control program) which define expected outcomes and which effect subsystem progress towards those outcomes. The problems of the origin of goals are either ignored or they are referred to as somehow "given." Man-made computers, from which many such psychological models arise, contain either implicit structural goals or explicit goals which are pre-fabricated. But in an evolving system, where do the goals and the system of goal construction and satisfaction originate? The geneticists' argument invariably must reduce to an assumption that random events gave rise to a first chemical feedback system whose products increased the probability that it would be reconstructed (reproduced), presumably by limiting the following chemical organizations.

Given that a feedback system spontaneously occurred, can the feedback concept account for a teleonomic mechanism? To accomplish this, feedback would have to provide a mechanistic interpretation of final cause. The minimal bio-psychological functional unit in a teleonomic system would be the negative feedback loop. In such a system, the loop is a mechanism subject to the physical properties of the nervous system and consequently must be state-determined.[5] But the physical reference signals of this nervous system represent various controlled quantities or goals -- or, if you will, purposes. The aimed-at future is, in a sense, represented by the reference signal. In that it attempts to achieve a condition in which referent is isomorphic with actual state, the system is future-driven. But it is mechanistic. Effect does not control cause, but the difference between the values of a reference signal representing a desired effect or goal and a signal representing actual conditions does determine what adjustments (e.g., changes in overt behaviors, perceptions, or cognitions) will be made. In most cases these adjustments cause the differences to decrease.

This description of an organismic teleonomic system based upon feedback has been attacked as scientifically untenable. Taylor (1950), criticizing a paper by Rosenblueth, Wiener, and Bigelow (1943), suggests that an event determined by feedback can not be operationally distinguished from other predictable events. Scheffler (1966) attempts to solve the problem of what Taylor calls a "missing goal object" external to the organism. Taylor notes that since any goals can be selected by the observer to describe the reference condition of an organism, the notion of goals is unworkable. If the goal is an attribute of the organism, not of the environment, how does one ascertain when it has been attained? Scheffler re-interprets the relations between goal, discrepancy, and corrective action as relations between semantic "inscriptions": desiring true, believing true, and making true. He suggests that these terms referring to desire, belief, and performance avoid implications of external goals. Unfortunately Scheffler's "solution" reifies exactly what the notion of feedback was intended to eliminate: the occurrence of untestable references in explanation. The argument, then, is that feedback is incapable of operational distinction from open loop systems because it refers only to internal interpretative events. An accompanying criticism must be one of the constructivist position. Since observable stimuli are no longer identifiable as properties of an environment external to the organism, how can the psychologist determine the states of the environment input relevant to his subject and predict their influences on behavior?

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[5] This account assumes that a nervous system is a defensible construct and that statements about its feedback loops also describe psychological feedback loops in which goals and actual conditions are represented by neural states.

W. T. Powers suggests that the experimenter with a constructivist epistemology should be interested in identifying the "controlled quantity," "...an environmental variable corresponding to the perceptual signal in a control system" (1973, p. 283-284), its relation to a reference condition, and outputs resulting from detected errors. Neither the perceptual signal nor the reference signal can be directly measured since they are constructs said to be internal to the organism. But changes in what the experimenter calls "environment conditions" which may be "controlled quantities" should cause equal and opposite changes in the system's outputs. Both environment conditions and behavioral outputs are observable by the experimenter (they are previously verified constructs). When alterations of environment conditions result in no change in the organism's behavior, the hypothesized controlled quantities can be rejected. Feedback systems and their controlled quantities are identifiable. But what of the rubberband cited earlier in a condemnation of the feedback model? How can its repeated returns to a single state after it has been stretched be contrasted with feedback rather than described by feedback? Construct a feedback experiment (a "Test for the Controlled Quantity," Powers, p. 232): The observable environment condition is varied; a force is exerted on the band and it begins to stretch. Our hypothesis is that the length of the rubberband is a controlled quantity. If it is correct, an observable force (output or behavior) equal and opposite to the original force exerted by the pulling would become evident and the band length would remain invariant. In actuality, the band does stretch, returning to its initial condition only after the disturbing force is withdrawn. The length of a rubberband is not the controlled quantity of a negative feedback loop. It is an element of an open loop in which only unidirectional actions and reactions occur.

Conclusion

Can radical constructivism as a philosophical vantage point be used to generate viable models in psychology? To answer the question, one must specify potential mechanisms by which the organism constructs. It has been proposed that such a system operates by coordinating its own actions, with the intent of generating and testing conceptual invariants. "Testing" was conceptualized here as the operation of a negative feedback loop. "Generating" has been treated in a description of converging operations. Some of the resulting invariants compose what the realist calls an "environment."

The epistemic view of the constructivist, the discovery of converging operations or "invariants," and the application and testing of operational constructs via feedback all seem to be interdependent. When two or more actions (including perceptions and concepts) repeatedly result in a single novel perception or conception, a third construct is synthesized. Seeing a configuration and touching it yields one concept of object. Sucking and moving the head can yield a perception of milk and become a single construct, called "search." Finding that two methods of measurement result in identical outcomes allows for the creation of a scientific concept ultimately defined by the measurement activities. These syntheses reflect the applicability of operationism to constructivism in psychology. The constructed invariants (e.g., knowledge of objects such as "car," of activities such as "searching," of scientific concepts such as "momentum") are applied and tested by the organism-scientist in reducing discrepancies between desires and expectancies (e.g., avoiding collisions, eating, explaining the interaction of moving objects) and actualities (e.g., colliding, hunger, an inability to explain). The process of detecting discrepancies is called negative feedback. This process is the recursive element of a closed-loop system which also contains operations by which discrepancies are reduced. The operations are themselves constructed by a developmental feedback process which identifies convergences.

The organism-scientist does not exist in an environment which can be defined apart from his actions upon it. Rather, the environment is the result of his activity and exists in him. If one accepts this premise, the scientist looking for causal explanations of behavior must search for these causes within the organism and nowhere else.

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In the Garden of Choices[1]

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One of the difficulties every teacher runs into, it seems to me, is that of reconciling a number of divergent goals, none of which should be neglected. For instance, it is important that the child - and above all the small child - be given a framework of certain knowledge. At the same time, however, the child must not be turned into a dogmatist, nor should his critical and creative participation in the schooling process be diminished. Similarly, it is important that the child derive from his schooling an ethical orientation that is both social and individual, yet, at the same time, he is to be left flexible and autonomous in his choice of attitudes and in the formation of his values. In both cases the child has to be steered clear of dogmatism on the one side and of scepticism on the other.

The difficulty of harmonizing these divergent goals, fortunately, does not spring from the pupil, at least if he is taken at an early age. Small children are remarkably pliable, curious, and responsive. Rather, the difficulty springs from the manner in which several subject matters are traditionally presented.

I propose to divide the subject matters into three types: first, the subjects that deal with physical and "psychological" things; second, those that deal with mental things;[2] third, placed in opposition to the other two areas, history and geography. The distinction is based on whether or not the area offers the possibility of experimentation.

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[1] Translation by Ernst von Glasersfeld, approved by the author.

[2] Ceccato's terms for the three types are: fisico (physical), psichico (psychical), and mentale (mental). No substitution of English words could convey Ceccato's meaning which is central in his operationalism or constructivism. In the context of this essay it may suffice to say that "physical" refers to items that have been given a location in space, "psychological" to items that have been given temporal location only, while "mental" refers to items constituted by specific operations of the mind regardless of the physical or psychological items they may be applied to. For a more extensive exposition of this classification, the reader is referred to Ceccato, Un tecnico fra i filosofi, volume 2, Padua: Marsilio, 1966; pp. 20-54. (Translater's note)

It has been known for a long time that the very activity of experimenting is sufficient to prevent the learner from assimilating passively, from becoming a dogmatist or a skeptic. These three negative aspects are, in fact, rather rare in technicians.[3] He who runs an experiment, once he has found or organized things in a certain way, expects to rely on the outcome of his activity and is, indeed, ready to accept it either as confirmation or as disconfirmation, as verification or as falsification of prior experience, of his hypotheses, or at least as suggestion of a modification of these which thus amplifies them.

What, however, are the conditions necessary for experimentation? One of them is well known: there must be variables, parameters, and one of them must be left free, i.e., the one that is put in question, while the others are taken to be certain and fixed. But there is also another condition: one has to have a criterion by means of which one can construct or recognize in a univocal fashion the things with which one is dealing. If the criterion varies, the results, too, will be variable; and if there is no criterion, there will be no results; indeed, if there are no criteria, no experiment can take place.

In the physical sciences these conditions are amply satisfied and, with some limitations, they are satisfied also in the psychological sciences.

In history and geography, at least with regard to what has not yet happened or has not yet been explored, the wait-and-see attitude should characterize the student. There may, of course, be dogmatic or skeptical historians and geographers, but if they have these attitudes, it is not the fault of their disciplines nor of the particular situation in which these disciplines may be at the moment.

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[3] This may be so for European experimentalists; a survey of experimental psychology in the United States would hardly support this assertion. (Translator's note)

In the disciplines that deal with mental things, however, we get a very different picture. To put it bluntly, these topics have been traditionally shaped in such a way as to exclude experimentation. This is so, because at least some of the constructs they involve are expressed in terms that are metaphorical, or negative, or tautological, or have a range that is so general that they fit anything whatever. Besides, some sectors of this area have established an axiomatic type of presentation which, even if it did employ proper or positive terms, would inhibit live experimentation because they reduce freedom to the choice of pieces and rules of a ready-made game.

The mental disciplines are in fact rich in alternatives and choices, and therefore they should be more conducive to the preservation of creativity and the development of responsibility (which we know well enough, since it is easier to change one's mind or one's attitude than to change the course of the stars). Yet, as a result of the way in which they are traditionally presented, it is precisely these disciplines which appear as the most rigid ones and least separable from the scholastic-academic context. One of the reasons for this is that whatever is designated by a metaphor or a negative term (which, if analyzed, turns out to be contradictory or circular), remains enveloped in a kind of magical cloud and creates the impression of transcending the comprehension of the learner as well as of the teacher. This leads to embarrassment and frustration and flight into aggressivity, fuelled by the terror of touching or changing items that rest on nothing but literal memory.

Listening to any examination, it is easy to see whether the candidate answers on the basis of actually performed operations, operations which he has made his own and is now applying, or on the basis of verbal memory.

The difference becomes clear if one, first, asks questions such as, "What is a name?", "What is this or that grammatical category?", "What relationship obtains between a line and a surface?", and then asks questions such as, "What is milk?", "What is the relationship between the movement of arms and legs when you swim a breast stroke?", etc. Questions concerning grammar and syntax, elements of mathematics, and the like, have a specific effect on the child or juvenile: the muscles in his neck are tensed, the gaze is fixed, he speaks without inflexion; and the questioner often hides behind a mask of intransigence or superiority. But when talking and operating have worked hand in hand from the beginning, the examinee relaxes and tends to overcome his uncertainty because he reconstructs what he himself has constructed before. This, by the way, corresponds to the word "know," which in everyday usage (but not in the metaphorical use of the philosopher) signifies the capability to replicate something that has previously been made and recorded.

This distinction I consider very important. It is based on the difference between two types of object and between the two traditionally opposite ways of speaking about them. On the one side there are words and actions that reflect one another; on the other, there are words, but they engender no activity. It should be easy to recognize in this the old distinction between the concrete and the abstract sciences and also that between science and philosophy. My distinction, however, springs from reasons and assessments that are not at all the traditional ones.

* * *

I shall repeat here a number of considerations that I have expounded many times in contexts of critique as well as of construction. I beg the reader's forgiveness. But I believe that these considerations are necessary to justify the introduction of elements in the area of education, elements which are novel though they may well have been awaited for a long time.

First of all, let me state the principles of instruction:

a) The active participation of the pupil;

b) The interaction between school and life, between school and home, between the voice of the teacher and many other channels;

c) A content that does not overtax the capacity of the pupil (Paul Claudel lets one of his characters say: "What I don't understand I consider a personal insult.");

d) The circular arrangement of the various disciplines (conventionally separated but all reducible to the unity of the operating mind);

e) Contents that are self-developing and act as terms of comparison, both critically and constructively, with regard to what will be faced, once school is finished (i.e., an education that is permanent and recursive and, besides, has a relevance to both cultural and practical life).

They are difficult principles to live up to. They are difficult, even for the sciences we call concrete or natural, if even their knowledge is taken to be a passive reflection of a nature, or reality, that subsists in itself, ready-made and structured, such as it appears in our thought and language. That, in fact, would imply that only one of all possible constructs (knowledge) could be deemed true, universal, and necessary, while all the others would be false, arbitrary, personal, etc. Yet, there would be no criterion whatever to effect the comparison with nature, with transcendent reality, in order to separate the false from the true.

Whatever the conception of knowledge, experimentation, as we have seen, is compatible with the above principles. But from the disciplines dealing with mental objects, experimentation is banned.

Let me repeat also the reasons for this unfortunate situation in the mental disciplines.

Some twenty-five centuries ago, as far as we know, when the interest and curiosity of researchers spread from the physical world to that of the mind, the methods of investigation used in the first were transferred to the second. That is to say, any investigation had to involve several percepts (at least two) both spatially localized and put in relation with one another.

One of the first objects to be studied was the single act of perception, and, unfortunately, a single percept does not fit that schema -- it contradicts it. When a fit was forced, the percept had to be made into two; i.e., it had to be doubled. Thus, perception came to be visualized as a relation between two percepts, both equal, as though one were the reflection of the other; and both were located in space, one outside, the other inside the perceiver's body.

Of course, problems and difficulties arose at once. How, for instance, was this doubling to have taken place and how was a comparison between the two items to be made, if the one was outside and unknown, while the other was inside and the only one known? Also, how could the inside one be equal to the outside one, if the space in our heads was already taken up by other things such as the brain?

The history of philosophy as well as the introductory part of the natural sciences (i.e., their pre-physical and pre-psychological part, those aspects that are called methodological or logical because of their elements, and what they are founded on) were tainted with this misconception and they can be considered a collection of efforts to cover up the original mistake of doubling the percept, keeping it alive nevertheless at their roots. It is this mistake, indeed, that forms the constitutive act of all philosophizing.

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The impossibility of describing a mental object in its own positive terms, and thus to trace the designated activities from the words, this impossibility derives from the original mistake.

* * *

The paths that have been adopted to escape from the embarrassment caused by this mistake, have on the whole been three.

First of all, no one risks entering the field without indicting another scholar and sometimes more than one. In other words, one joins battle with colleagues and no longer directly with the object to be investigated. One selects and criticizes one among the many difficulties that have been created by the illicit doubling of the percept, and then offers one's own cover-up. The history of the cover-up, naturally, gets more and more involved: "So-and-so has said this, so-and-so the other, etc."

One of the fundamental points in the history of the doubled percept was, for instance, the introduction of the "concept," or "idea," or "universal," etc., not in the usual sense of these words (which allude to what one knows, i.e., to thoughts that take place around a construct) but, instead, as the "internal" counterpart of an "external" object, a counterpart which becomes the necessary mediator between the things named and their names. But what relation could such a "concept" or "idea" possibly have with the thing that corresponds to it? Does it come first, _ante_ _rem_? Does it come after it, _post_ _rem_? Is it inherent in it, _in_ _re_? Is it of earthly or of heavenly origin? And so on. Every author will propose his own answer. And thus we get the history of philosophy and a history of every mental object - histories from which it is considered impossible to get away.

If men have occupied themselves with a specific thing for a certain length of time, one can certainly write a history of this interest of theirs.

This could be the history of the automobile, of water, or of any other specific item. But if we want to investigate or in any way deal with an automobile or with water, we can very well start from the situation as we find it, taking the item and the theories and practices in which it is involved as they are at the moment. In other words, we can start from the thing, without necessarily going back to its origins and without considering it historically.

The function of the philosophical histories, however, is quite clear. For the moment they dispel embarrassement, even if in the long run they are going to increase it. Which, among so many voices, shall be the right one? In fact, those who fear a loss of faith and want to avoid discouragement or skepticism have often suggested that it is better to study this history as little as possible - and in this way they have substituted dogmatism for what has been called problematicism.

The second path consists in eliminating the soft points from one's treatise. This can be done if the treatise need not have a value in its own right, i.e., as philosophical work, as entry in an academic contest, as preface to a technical-scientific study in a specialized branch, etc. It is the solution which, for instance, Newton adopted with regard to space and time, when he declared that he would not define them, quae notissima sunt.

Today we are witnessing the disappearance of definitions from the primers and from the textbooks of the lower grades in general. They are mostly replaced by examples of what is being talked about, particularly with regard to notions of linguistics, of mathematics, etc.

Where encyclopedias are concerned, even those that are "encyclopedic" and bulky, unless they are explicitly destined for the philosopher, they prefer the ignorant solution. Thus, the Britannica, under the entry "Thought,"

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refers the reader to "Laws of thought," and in the Treccani[4] "Thought" is not an entry at all.

The third and last escape route from the difficulties created by mental objects as a result of the original mistake, is that of embedding the names of these objects in a network of expressions that have no immediate ties with the mental objects themselves. The mental objects are then supposed to acquire their characterization through the relations which these expressions posit between them. This is the way in which the famous axioms were born, venerable and unalterable, because they cannot possibly be changed by any investigation of objects. The survival of an axiom, however, is due not to its venerability, but rather to the fact that the expressions with which it becomes enmeshed are irreducibly metaphorical, negative, or contradictory.

Obviously none of the three paths followed by traditional instruction with regard to mental items achieves the intended goal we spoke of; i.e., of offering to the juvenile or child a framework of certain knowledge without at the same time turning him into a dogmatist or simply precluding him from becoming creative and critical about his learning.

* * *

But is there a fourth path? I believe the answer could be affirmative. The path is there, even if it is not yet completely cleared. What I have in mind is a method of instruction that exploits the results that have been obtained in the course of studies aiming at the construction of a mental machine, a machine, that is, that perceives and represents things to itself, that categorizes and thinks and, finally, speaks. In order to design and build such a machine, it is indispensable that for every discourse there be a corresponding dynamism (activity) that can be articulated into ever more minute operations, down to those minimal operations to which, then, the functioning of organs can be made to correspond. These organs, moreover, have to be such that they can actually be produced by present-day technology.

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[4] The oldest and most prestigious Italian encyclopedia. (Translator's note)

These studies are being carried out by a branch of modelling cybernetics, which we call "logonics," while "bionics" is the branch inspired by findings in the natural-physical sciences. One advantage of these studies is that they compel the researcher, on the one hand, to take nothing for granted, to consider nothing "notissimum," and on the other, to check, in the functioning of the model, the correctness of his analysis and description of the operations performed by man. Lastly, if these descriptions do, indeed, serve as blueprints for an actual construction, one can be quite sure that they do not contain metaphors, negations, contradictions, etc.

Instead of going into the philosophical history of a notion one wants to communicate, and instead of leaving the formation of the notion to the intuition of the student and the teacher, one will indicate the operations that constitute that notion and one will try to make sure that the student, while executing these operations becomes aware of them, learns to control them, and in this sense becomes an experimenter.

* * *

To explain the application of this "operational" method of instruction, I shall take three examples of grammatical terminology that are usually introduced fairly early in school: "singular," "plural," and "collective," as used in the classification of nouns. They could be treated as pure categories, i.e., apart from the nouns to which they are attributed. In the case of the first two, this could lead back all the way to Parmenides and Heraclitus.

In the case of the third, it would lead to a term that is fundamental in so-called modern mathematics, namely to the famous "set." A school grammar obviously disregards these problems. It avoids them by taking into account only the linguistic presence of the items, for instance in examples such as "leaf," "leaves," and "foliage."

Someone who has already learned to use these words, carries out the operations they designate and is, therefore, quite able to carry them out by themselves, whenever he uses the words "singular," "plural," and "collective," just as he will carry them out whenever they are implicitly applied to any other mental construct such as "shrub," "shrubs," and "shrubbery," or "Crock," "crocks," and "crockery." The grammar book, thus, can present an open list of words under each of the three grammatical terms, leaving to the child the task of constructing their designata unconsciously. This is no different from what the infant does, who, once he has learned to pick up some object in his hand, repeats his gesture either in the air or in picking up some other object. But in both instances, the operations that are being acquired, are acquired unconsciously. This, among other things, creates the impression that what happens has to happen in that way, inevitably and universally, and that there are no alternatives. Above all, it makes impossible any awareness as to the consequences it might have if one considers the same physical or psychological object in the three different ways. Hence the grammarian feels obliged to come up with an explanation.

Inevitably there will be a passage saying something to the effect that "nouns indicating one thing only, or one animal only, are singular in number, and nouns indicating two or more things, or two or more animals, are plural."

The weakness of these "definitions" is obvious enough, even if we overlook the fact that usually no attempt is made to make clear what kind of an item a noun or a name is. The definition, in fact, tries to explain something that is simple by something that is more complex. The item that was to be explained, thus, not only remains unanalyzed but also illicitly enriched, i.e., more complex than it actually is. The singular is presented either as an applied singular (i.e., some word in the singular form) or with the addition of the meaning of the indefinite article or the numeral "one," of "only" or "single," or some other word that has a proper meaning in its own right. In short, one comes to identify "tree" with "a tree" or with "one tree," or even with "a tree alone" or "one tree alone."

Obviously, anyone who might want to retrieve from one of these expressions the operations that constitute the item we call "singular," will be led astray. The "only," for instance, requires a repetition, in the sense that the operations carried out are once applied to "thing" and then a second time in a pure state of attention, in suspension that is, without being focused.[5] The operations indicated by the indefinite article we shall discuss below. The number one, moreover, requires the repetition of the operations that constitute the singular. In any case, the surplus of operations that are brought in by explanatory expressions of the kind quoted above, become hopelessly and illegitimately entangled with the operations that constitute the singular, which was the item to be defined.

In the case of the plural or the collective the situation is much the same: instead of defining what is to be defined, more undefined items are added.

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[5] Ceccato's use of the term "attention" is more complex than it appears. In his view, attention is not merely a passive state but it is active and dynamic. It can focus not only on other items but also on itself, in which latter case it gives origin to what he calls "mental categories." (Translator's note)

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The analyses we have carried out with a view to building a model of the mind, as well as my didactic experiments, suggest a different approach to the notions of singular, plural, and collective. It is an approach that can deal with all notions whose content is mental.

It is good practice always to use something that, while remaining unchanged on the physical level, can be categorized in different ways. I have often made use of a drawing that can easily be called either "forest" (applying the singular), or "trees" (applying the plural). The child quickly understands that the difference cannot spring from the drawing, i.e., from something that imposes itself by itself. And that already helps to keep away the presupposition that a percept imposes itself by itself and has to be doubled. Instead, the child's attention is shifted to what he himself does, on the operating of his own mind.

A detailed analysis of the operations involved has been carried out (Ceccato, 1969, p. 52-53). Here we need only say that even a child quickly realizes that when he categorizes the drawing as a singular, his attention is detached (defocalized) before and after the design, whereas his attention is attached (focused) while he is scanning the design. When he categorizes the drawing as a plural, on the other hand, his operating is inverted; i.e., he detaches his attention in the middle, whereas he focuses it on the design both before and after this moment of detachment.

A drawing of a tree in full leaf may serve to bring out individually the operations that constitute the categorization as collective, because here, again, there is the possibility of speaking either of "leaves" or of "foliage." With the plural, as we have seen, the design is not isolated, or closed, but instead remains open; i.e., attention is focused on two sides. Now, in the case of the collective, it is not difficult to realize that what was left open in the plural now is closed by the categorization as singular, i.e., by the application of the category of singular.

The drawing of a tree also helps to discover the difference between the operations designated by the collective "foliage" as opposed to those designated by the pseudo-collective "crown" which, too, can be considered a composite consisting of elements, i.e., of leaves and branches.

We can now specify the operations that are the inverse of those involved in the collective, i.e., the operations that lead to categorizing something as an element. These operations are linguistically designated - at least in Italian and English - by the indefinite article added to a noun. The construction consists in inserting the item that is to be considered an element into a plurality. That a plurality is required can best be demonstrated by performing the operations designated by the indefinite article in two instances that involve different items. First with an item such as "'a' leaf," i.e., an item that can be pluralized simply by attributing different locations (in space or time) to its repetitions. In the case of "a leaf," however, it is easy to overlook the fact that one has to have several (i.e., a plurality of) leaves in order to speak of a leaf. This pluralization becomes more obvious with the type of item that can be pluralized only by attributing a difference of color, of material, etc., to each repetition. For instance, "'a' red" or "'a' water," in which case it becomes very clear that, in order to accomplish the operations designated by the "a," we have to imagine a plurality of different reds or different waters, i.e., items that differ not only in location but also in a characteristic.

In this way one can reach sufficient awareness in order to analyze operationally the definite article "the." If plurality characterizes the indefinite "a" and hence also choice and discarding, novelty, etc., the definite

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article "the" designates the picking up of an already made item, i.e., something that could not be anything else.

* * *

I know what objections this approach to instruction and even to research raises. Here I should mention at least two. The first concerns the accuracy of the operational analyses of mental items. I certainly could not say that I am absolutely certain of their correctness, in spite of the fact that, for some of them, several other people have come to conclusions that are identical or extremely similar. But this margin of uncertainty is also an advantage: it stimulates the person who is presented with them to check them by investigating them for himself and thus to take the first steps towards live and also exciting experimentation.

The important thing is to show the child the direction in which to go, to teach him to find his own path, to retrace it, and to continue it. Only in this way will he be able to assume a scientific attitude with which he can approach also the things of the mind.

The second objection stems from a particular way of judging the juvenile and, even more so, the child. Those definitional problems are not supposed to be suitable, it is said they are problems for adults and cannot be broached before the end of high school, or college, or even graduate school. This would be true if and only if the notions were approached from a philosophical point of view. Experience, however, has convinced me that as soon as the philosophical approach is replaced with an analysis in operations, the child appropriates with great ease the required mental mechanisms, provided that they are illustrated by a sufficient number of examples. If, in the case of singular and plural the tree/forest example, should not be enough, there is no shortage of others (claps of the hand/applause, waves/

choppy sea, rain drops/rain, etc.). More efficacious even may be the suggestion to carry out the operations that constitute *singular* or *plural* while whistling, accompanying it first with the word "whistle," and then with "whistles." As a rule, the subject will become aware of how he structures the same continuous sound as a single item in the first case (in response to the singular "whistle") and as a repetition of items in the second case (in response to the plural).

In any case, if the philosophical approach, be it explicit or implicit, is assimilated by the child during the first years of school, it will be a good deal more difficult for the youth, let alone the adult, to get rid of it in order to discover an alternative. Once one defines in metaphorical, negative, tautological, etc., terms, one does not only define badly certain things, but one gets used to defining badly, and one will go on to define any other thing in those same indefinite terms and, finally, one will come to consider this type of definition obvious.

I emphasize: as far as the teacher is concerned, what is needed is only patience and a little fantasy. Indeed, I am convinced that much more can be done to give the child an awareness of mental operations, though my experience is not sufficient in that regard. I have tried to suggest that the problem is difficult only if it is presented in the traditional philosophical key, not if it is presented in the operational key, i.e., by stressing and explicating the subject's own constructive operating. In fact, the moment you think of philosophy, names and problems and statements keep cropping up that have racked the brains of many a schoolboy and college student. The reason springs always from the same root: the original illicit doubling of the percept and the insoluble problems deriving from it.

Once the systematic error has been spotted, not only mental activity becomes accessible to analysis without too many obstacles, but it also becomes relatively easy to step out of the philosophical tradition. And once we can do that, it quickly becomes clear how the abstruse expressions of the philosopher have arisen, be it those that belong strictly to his own realm or those that he loaned out as a foundation to other disciplines. The result of this detachment is a history of philosophy that acquires something of a "whodunnit," a sort of chase after the error, apt to remind us of human fallibility.

For the growing child it would be something like a vaccination.

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Constructivism and Principles for Instruction[1]

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> It is not that they can't see the solution -- they can't see the problem.
>
> (Chesterton)

> Science is not just a collection of laws, a catalogue of unrelated facts. It is a creation of the human mind, with its freely invented ideas and concepts....The only justification for our mental structures is whether and in what way our theories form...a link with the world of sense impression.
>
> (Einstein)

> The drive to lead an intellectually satisfying life is...a product of long process of education...(and)...is an autocatalytic affair, growing with the practice of it.
>
> (Bridgman)

Everybody "knows" what is wrong with education, in fact the definitions of the problems by professionals are the same definitions of "everyman." And, in most cases, the solutions proffered by professionals are not only the traditional ones but often more trivial than those of the "amateur." Few have the courage to face two fundamental questions involved: 1) what is the purpose of the institution of public education, and 2) what is the nature of knowledge. Failure to deal with the first leads to tinkering with trivial elements of a complex societal institution that should be a mechanism for guiding change (whether of the child or society) while unwillingness to confront the second produces, at best, temporary excitement about "innovation" and, of course, keeps thousands of people in higher education and the federal government busy.

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[1] The major ideas in this paper were first presented at a Symposium on Mathematics Education at Northwestern University (February 15-16, 1973).

There is hope, however. A few are beginning to view the school as an important element in cultural change (e.g., the Follow Through Program, USOE, 1973 and Sarason, 1973). The purpose of this essay relates to the second point and explicates certain issues relevant to implications of one's view of the nature of knowledge for instruction of children.

Currently, either the name Piaget or the term laboratory, especially mathematics laboratory, is sure to attract the interest of educators. While "laboratories" are gaining in fad appeal, meaning for such an instructional technique has remained only loosely and ambiguously defined. Teachers who attempt to use such a teaching strategy typically have been forced to rely on a set of "rule of thumb" slogans such as: "concrete understanding before abstract"; "intuitive understanding before formalization"; "activities, then pictures, then symbols"; "discovery rather than reception methods"; etc. All such slogans refer to distinct instructional variables which often specify contradictory approaches to teaching if their range of appropriateness is not qualified and coordinated by at least an embryonic theory of instruction.

Educators appear to have little interest in theory construction despite the current interest in "model" building. Theory, too often, is used to justify instructional biases and the "laboratory" and/or "open-classroom" is considered only an instructional device. Theory construction is necessary to help clarify the different types of roles that different classroom structures have in educational research and practice. Actually, a classroom laboratory should provide a context for research on problems relevant to specific aspects of the instructional process (variation of teaching strategies and techniques) and for discovering those psychological conditions critical for development of thinking in children.

However, in my opinion, Chesterton's remark is quite appropriate: "It is not that they can't see the solution -- they can't see the problem." We have yet to identify the fundamental dimensions of educational and instructional problems facing the educator. Key psychological principles for mathematics instruction and construction of a theory of instruction can be realized only after this first step has been achieved. A body of knowledge now exists in developmental-cognitive psychology that should have considerable utility for contributing to more refined theories of and strategies for instruction; i.e., for creating better school learning environments. One approach to the immediate task, then, is to search for suggestions from developmental psychology that pose relevant problems for developing theories of instruction.

There is no paucity of choices of psychological "models" available from which to start the search. Berlyne, Bandura, Bruner, Scandura, Skinner, Suppes -- and of course Piaget. Each has proposed a set of ideas (relevant to a theory of instruction) requiring theoretical and empirical study. The selection of any one model, however, brings with it many hidden presuppositions and is determined in no little part by one's own preconceived notion regarding human development, learning and education. As a group, these models represent a virtual wonderland of exciting ideas. Each of us can understand Alice's dilemma better as we explore their fantasies (Suppes, 1972).

A theory of instruction must begin with an adequate theory of cognitive development and learning. No longer can we accept that statement as "obvious" -- and go about the business of generating a multitude of methods based on unorganized intuitive rules based on inadequate knowledge of the process of cognitive development in children. All educators need to return to the beginning and ask, not "how do we teach?", but rather "how do children learn?"

Modern developmental psychology provides a necessary, but not sufficient, body of know-how for identifying some of the fundamental issues, constraints, and facts associated with the process of generating a theory of learning and instruction. But, to imply and act as if psychology had become relevant to mathematics learning only AP (i.e., after Piaget), misses a fundamental point about the relation of the science of psychology to the science of education, and distorts the history of both. On the other hand, I do not want Piaget's theory of cognitive development abandoned without clear understanding of why. The historical pattern in education and psychology seems to be one of enthusiastic adherence to a relatively novel theory -- with disappointment and rejection following close behind. The absence of serious controversial issues underlying much of the current research in cognitive development increases my concern that much which is valuable in Piaget's theory may be lost.

Natural Genetic Epistemology and Cognitive Development

Many psychologists, including myself, consider Piaget's clarification of the necessary bases of theory construction as important as his cognitive developmental theory *per se*. Implicitly, and explicitly, Piaget was greatly influenced by advances in theoretical physics (Bridgman, 1927) during the 1920's and 30's. The fundamental aspects of relativity theory which cannot be ignored in psychological theorizing is that conceptual judgments are always relative to the position of the observer and that analysis of knowledge acquisition requires a description of its operational basis; i.e., the mental operations of the individual that are associated with the construction and maintenance of consistent patterns (structure) of his continually transforming relations with the physical and social environments. Thus Piaget is unique in that his emphasis on a constructivistic theory of knowledge (Piaget, 1968, 1971a) is indissoluble from his interpretation of operationism -- that is, the need for operational analysis of the process of knowledge acquisition. "Reality" is constructed, not imminent in mind, man or stimulus, and applies to the child and, contrary to some interpretations (e.g., Stevens, 1935), theorist alike.

The form of epistemology typical of American psychologists (cf: Mischel, 1971) has been naive realism, and that orientation has been quite useful. But, it should now be obvious that our epistemological preconceptions, whatever they may be, are part of our view of the child. Kessen (1966) states the issue clearly: "The child who is confronted by a stable reality that can be described adequately in the language of contemporary physics, is child very different from the one who is seen facing phenomenal disorder from which he must construct a coherent view of reality" (pp. 58-59).

Analysis of cognitive learning and development, then, is always "biased" by the fact of a context of preconceived ideas of reality (i.e., western culture) and a particular set of concepts or theory and selected observations. Piaget's approach to the analysis of the development of children's conception of space provides us an excellent example (Piaget and Inhelder, 1956). The specification of a conception of space toward which the child will most likely develop, i.e., that conception held by most adults, is the critical first step. Observations and interpretations of the child's behavior are organized around the specifications inherent in that "endpoint" of development. Observations not congruent with, and not structurable in terms of the specified endpoint characteristics, no matter how reliable, cannot be considered a relevant part of development. This is not an example of bad science of inappropriate procedures but rather illustrates that conclusions about the child's cognitive structure is often more a function of the construction of reality imposed on him by the scientists rather than of the reliability and generality of "objective" observations. Whether we are engaged in instructional practice, research, or theory building, there is, for each of us, a set of guiding propositions that constitutes a theory of learning and development. These "fantasies" or "freely invented ideas and concepts" provide a particular coherent view of the developing child and of the presumed determinants of learning.

Piaget's constructivistic epistemology, and his biological background, predisposed him toward an operational and structural analysis of the knowledge acquisition process (Piaget, 1967; 1968; 1970a; 1970b; 1971a). The essentials of his position require only brief review there. Knowledge is defined as *invariance under transformation* (a most familiar concept to mathematicians). The construction of invariances in organism-environment relations takes place through the *operation* of two complementary biological adaptation processes, both of which are under the control of the internal self-regulating mechanism of equilibration.

One of the two processes (assimilation) concerns the application of cognitive operational systems (structures) to the organization of sensory data. New data or events are incorporated into existing structures through both on-going *physical* and *mental* activity. Such events and the products of new experience can be incorporated into the cognitive structure only to the extent they are consistent with existing functional structures. Accommodation is the complementary process whereby adaptation occurs by integration of existing structures with other functional structures and/or by differentiation of new structures under confrontation with new experience.

Activities such as play, practical or symbolic, represent assimilative activity; whereas memory, in the sense of invoking past experience, and imitation are accommodative since only prior formed structures are transformed for a new use or application. Assimilation is an active constructive process by which the data from experience are transformed and integrated with an already generalized cognitive structure. Accommodative activity, on the other hand, is the process whereby modified existing structures or novel structures are brought to bear on newly assimilated sensory data.

Too often instructional theory and practice have emphasized assimilation (i.e., "play") or accommodation (i.e., imitation) activity and neglected the role of equilibration of these complementary processes for cognitive learning. However, generalization of Piaget's ideas to instructional theory and practice is not simple and straightforward. The special meanings attributed to "logic," the role of the equilibration mechanism in constructing experiential data, and the distinction between operative and figurative thought are critical to such an endeavor.

Operational Structures and "Logic"

Piaget recently (1970b) elaborated his position that all human beings possess the same biological structures and functions that, in "exchange" with the common features of the nature world, generate mental (operational) structures and functions characteristic of each stage of development. Logical thought, in the Piagetian sense, then, is universal and of fundamental importance to an understanding of development and learning. But, whereas Chomsky maintains the human mind is "programmed" at birth with cognitive structures (i.e., mental representation of a universal grammar), Piaget accounts for the universality and stability of structures across cultures (Piaget, 1967; Goodnow, J., 1962; Goodnow, J. and Bethon, G., 1966; Greenfield, 1966; Maccoby, M. and Modiano, N., 1966) in terms of the self-regulation mechanism of equilibration. Thus, Piaget proposes that the mind at any point in development (i.e., life) is the unfinished product of continual self-construction (1971); i.e., "logical" processes are generative and not fixed. Structures are not preformed, but are self-regulatory, transformational systems with the functional factors in that construction being the processes of assimilation and accommodation.

Intelligence, the basis of knowledge acquisition, has then, two aspects: adaptation (with the complementary process of assimilation-accommodation under the self-regulatory mechanism of equilibration) and organization consisting of sets of mental operations that form the basis for maintaining invariance under transformation (i.e., knowledge). It follows from these considerations that there is an inherent logic to development; i.e., operational systems consist of elements and laws of combination of those elements that form a "logical" closed system. These mental structures are observable in the actions of the organism in its environment and, further, are describable in terms of formal or logico-mathematical structures. Genetic psychological analysis of these structures is a necessary prerequisite to an understanding of thought processes since there is "no structure without genesis, no genesis without structure."

During the sensori-motor period of development, action structures of the individual are revealed in practical groups, i.e., the observable, coordinated actions of the individual (Forman, 1973). During the pre-operational period, the child constructs representations (figurative structures) which do not have the operational property of reversibility. Piaget was able to identify operational structures with clear mathematical system properties in children between ages five and seven. The discovery of a resemblance between the structure of the mental action system (reasoning or thought) and mathematical structures (i.e., mathematical groups and lattices) had a profound effect on Piaget's thinking. Thought, it would appear, has the same, or similar properties, as mathematical group structures and both are governed by the same internal logic.[2]

The basic structuralistic approach of Piaget involves finding or creating logico-mathematical systems that describe the thought processes of an individual. Mathematical group and lattice theory are algebraic systems (Piaget, 1957a, 1957b; Flavell, 1963) which might well describe operational thought and Piaget has tried to take maximum advantage of this approach. To review the fundamentals: a mathematical group is a system consisting of a set of elements, together with an operation (law of combination), which yields the following system properties: (1) when applied to the elements of the set, the combinatorial operation will yield only elements of the set; no elements external to the set can be produced; (2) each set contains a neutral (or identity) element that, when combined with another element of the set, yields no change; (3) each element of the set has a inverse which, in combination with any other element, yields the neutral or identity element; (4) the combinatory operation (and its inverse) is associative; i.e., $[(n + m) + p = n + (m + p)]$. Piaget found it necessary to generate a "grouping" model with additional properties (i.e., both group and lattice properties) to describe the concrete operational structures. Most importantly, the properties of these psychological groupings are not derived from the properties of things, but from modes of acting upon things. Thus, the elements of psychological groups are, themselves, transformation that characterize the individual's operations as he acts upon sense data.

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[2] Piaget never has tried to find a mathematical "model" to "fit" the observed facts of behavior; rather the mathematical aspect of Piaget's theory is unique in that he assumes, somewhat a reminescent of Boole's "laws of thought," an identity between the inherent logic of thought processes and certain mathematical systems that have become formalized and "externalized," through inductive reasoning.

The revelations emerging from relativity theory require a constructivistic position with respect to the nature of knowledge; i.e., understanding of knowledge acquisition requires a description and characterization of the mental operational systems applied to the data of experience. Piaget's emphasis on structural analysis thus makes contact with the epistemological implications emerging from relativity theory; the biologists' emphasis on development as the formation, differentiation and hierarchical integration of functional (action) structures; and the mathematicians' emphasis on formalized systems that permit description of these structures. The task of the developmental psychologist is to describe the nature of action structures of the child at each point in development and, as much as possible, to formalize those descriptions in terms of logico-mathematical terms.

The classical "conservation" tasks, if administered appropriately, form one basis for generating observation of coordinated actions that appear to reflect such mental operational structures. The available evidence appears to support the possibility that such operational (mental) structures "exist" both in terms of replicability of hypothesized developmental trends and in appropriately designed training studies (cf: Beilin, 1971a). At the same time, neither psychological nor educational researchers have yet devoted sufficient attention to the problems of the validity (i.e., internal consistency) of the grouping structures (Clarey, 1971; Green, 1971), nor to the role of such structures in learning (Berlyne, 1965; Bruce, 1968; Inhelder and Sinclair, 1969), beyond these classical situations.

Only recently have educators, especially mathematics educators, become interested in Piaget's views of fundamental logico-mathematical relations, such as his ideas about the logical properties of number and space. Beilin (1971b) points out that philosophers of science generally have emphasized the desirability of isolating philosophical and logical issues from psychological matters. Psychologists, mathematicians and logicians generally have maintained this position with respect to Piaget. However, a significant part of his psychological theory has mathematical and logical content which cannot be ignored by either psychological (Leskow and Smock, 1970; Alonzo, 1970) or mathematics-learning researchers. Mathematics education researchers rightly should be directed to the analysis of the logical and mathematical validity of Piaget's system and to the correspondence between the characteristics of the psycho-logic systems and those logic structures derived from purely mathematical analysis. Recent work from Steffe's laboratory (e.g., Kidder, 1973; Johnson, M., 1971; Johnson, D., 1972) represents an excellent beginning in this direction.

Role of Experience and Equilibration

Experience is not a unique factor in development according to Piaget. Merely being exposed to particular environmental situations is conducive neither to cognitive activity nor to developmental change. Children may or may not make discoveries in the course of play, and watching a laboratory experiment (or conducting one) may or may not help a child acquire a particular concept. Equilibration is the central factor in sturctural change whether that refers to "stage" or to concept learning. Equilibration is the process of intrinsic (self) regulation that balances assimilatory and accommodatory processes, compensates for external and internal disturbances (internal or external to a particular structure) and makes possible the development of more complex, hierarchically integrated operational structures. "Rhythms, regulations, and operations, these are the three essential procedures of the self-regulation and self-conservation of structures; anyone is, of course, free to see in this the "real" composition of structures, or to invert the order by considering the operative mechanisms the source of origin, in an atemporal and quasi-Platonic form, and by deriving everything from these mechanisms. In any case, however, it will be necessary, at least with regard to the building up of new structures, to distinguish two levels of regulation. On the one level the regulation remains internal to the already formed or nearly completed structure and, thus, constitutes its self-regulation, leading to a state of equilibrium when this self-regulation is achieved. On the other level, the regulation plays a part in the building up of new structures and integrating them as sub-structures into larger ones." (Piaget, 1968 , Ernst von Glasersfeld, translator.)

Disequilibrium occurs as the child assimilates data from immediate experience into existing mental structures. As cognitive structures change to accommodate to the new experiential data, equilibrium is restored. The equilibration process is one of auto-regulation -- both of the transformations of data based on existing cognitive structures and of changes through accommodation. Thus, the child must be exposed to situations that are likely to "engage" the functional structures; i.e., he must be involved in a personal striving to understand or "accept" the task as a "problem."

A basic question for instructional theory and practice is: What are the processes and conditions that motivate and insure engagement or acceptance of the problem task by the child? The source of "interest" that promotes striving for problem solution is contingent on assimilative-accommodative activity but the specifics remain unclarified in Piaget's theory (cf: Mischel, 1971). Within a structuralist framework -- if a structures exists, it must function -- multiple cognitive structures provide a dimension of openness that make probable continual sources of disequilibrium from interaction of the internal operational and/or figurative structures activated, as well as by exchanges involving novel experience. And, despite lack of specifications, Piaget is quite explicit on his general position: "It is not necessary for us to have recourse to separate factors of motivation in order to explain learning; not because they don't intervene...but because they are included from the start in the concept of assimilation...to say that the subject is interested in a certain result or object thus _means_ that he assimilates it or anticipates an assimilation and to say that he needs it means _that he possesses schemas requiring its utilization_" (Piaget, 1959). In passing, it might be noted that natural or life-like contexts seem to provide excellent situations for promoting cognitive change. Unfortunately, too little empirical investigation has been oriented to questions of the natural environmental determinants of curiosity of children at various stages of development and with different experiential backgrounds; i.e., what children recognize as problematic, and what kinds of incongruities are sufficient to motivate change in concepts and/or beliefs.

Cognitive conflict, or the awareness of a momentary disequilibrium, generates a need to establish consistency (equilibrium) between the existing schemas and/or novel information and is motivation for cognitive activities. Both application of an existing schema, and the elaboration of new ones in the course of development, stem simply from the overriding need to make "sense" of present problems by fitting them coherently into schemas "learned" in the course of solving prior problems.

The notion that disturbances introduced into the child's systems of prior schemas lead to the adoption of a strategy for information processing is the fundamental difference between the equilibration and associationistic theories of learning (Piaget, 1957b). For associationistic theories of learning, "what is learned" depends on what is given from the outside (copy theory) and the motive that facilitates learning is an inner state of some sort or other. Equilibration theory holds, however, that learning is subservient to development; i.e., what is learned depends on what the learner can take from the given by means of the cognitive structures available to him. Further, cognitive disequilibrium (functional need) is what motivates learning; i.e., questions or felt lacunae arising from attempts to apply schemas to a "given" situation. The child, then, will take interest in what generates cognitive conflict; i.e., in what is conceived as an anomaly. If the task demands are so novel as to be unassimilable or so obvious as to require no mental work, the child will not be motivated.

After the period of sensori-motor development, equilibration becomes a process of compensating for "virtual" rather than actual disturbances. At the operational level, intrusions "can be imagined and anticipated by the subject in the form of the direct operations of the system -- the compensatory activities also will consist of imagining and anticipating the transformations to an inverse sense" (Piaget, 1957b). Further, there need be no external intrusions in order for the equilibration process to be activated. For example, the acquisition of conservation concepts is, in Piaget's view, "not supported by anything from the point of view of possible measurement or perception -- it is enforced by logical structuring much more than by experience" (Piaget, 1957b). It is the "internal factors of coherence -- the deductive activity of the subject himself" that is primary. Equilibration, then, as we noted earlier, is a response to internal conflict between conceptual schemas rather than a direct response to the character of environmental structure factors. Bridgman (1936) expresses the same idea when he states "we do not experience things; things are a construction of ours the function of which is to emphasize the resemblance between aspects of our present immediate experience and aspects of our past experience..."

Operative and Figurative Thought

A considerable amount of confusion concerning Piagetian theory and its implication for both research and instructional practice derives from a failure to consider the figurative and operative aspect of intellectual functioning. In general psychological terms, the distinction is between the selection, storage and retrieval, versus the coordination and transformation of information (Inhelder, et al., 1967). More specifically, the development of any sequence of psychological stages, *à la* Piaget, consists of an interactive process of equilibrating functional structures of the organism with sensory event-structures of the perceived environment. Piaget (1964) analyzes "experience" into the two components of "physical" and "logico-mathematical" and this distinction between a physical and logico mathematical experience is essential to the understanding of the growth of knowledge. Knowledge based on physical experience alone is knowledge of *static states of affairs*; if a child is wrong, it is easy to demonstrate that to him. While knowledge emerging from logico mathematical experience is knowledge of transformation of states and quite another matter. If a child is wrong in the first case, it is easy to demonstrate his error, but in the second it is difficult, if not impossible, to convincingly demonstrate, or even to get the child to accept verbal explanation of the "correct" answer (Richards, 1975, p. 56). For example, if a child fails to *align* the two endpoints when comparing length of sticks, it is quite easy to correct the mistake. If, however, he fails to display *transitive reasoning* in a task, one or two examples is not likely to "teach" him the concept.

Physical experiences, then, provide for the construction of the invariants (Figurative process) relevant to the properties of objects (i.e., states) through exchange with objects involving sensory mechanism. For example, one may touch something and it is hard, cold, hot, soft, supple, etc. Or one may see something -- an object is red, a diamond cutting glass, the shape of a banana, etc. The course of logico mathematical experience, however, is assumed to be abstractions (operative knowledge) from coordination of actions vis-a-vis representations of "objects," i.e., transformations of the "states" associated with series of discreet physical experiences. The critical difference is that logico-mathematical knowledge demands that a pair (or set) or physical objects not be defined by the temporal-spatial (perceptual) similarities, but rather by invariant relations among or between objects.

Figurative and operational processes, then represent two types of functional structures necessary to account for knowledge acquisition. Figurations are defined as those action schemata that apprehend, extract and/or reproduce aspects of the prior structured or organized physical and social environment. Such action schemata include components of perception, speech, imagery, and memory. Figurations and associated acts are based on physical, as contrasted to logico-mathematical, experience and constitute the "empirical" world; i.e., empirical truth is no more than the "representation of past experience in memory."

Operations, on the other hand, do not derive from abstractions from objects and specific events; rather, operational knowledge is derived by abstractions from coordinated actions relevant to those events. Thus, operations are those action schemata that construct "logical" transformations of "stages." Such "logical" systems of transformations operate either upon representations of events, or on the cognitive system's own logical operations, i.e., reflexive operations.

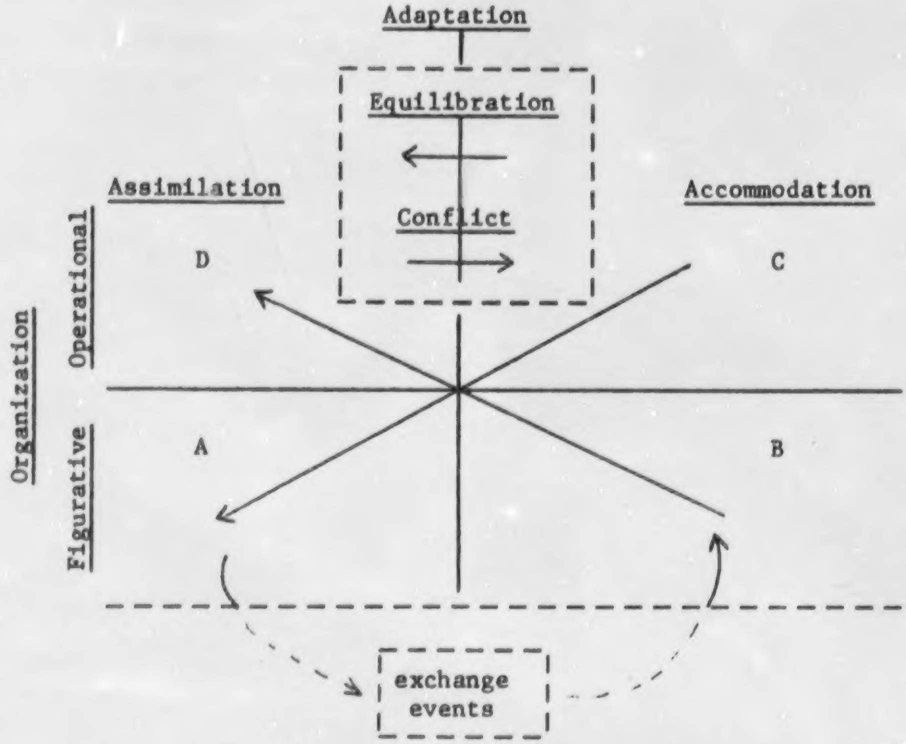
Figurative and operative structures are two parallel streams with their genetic or developmental origins in the same source (Piaget, 1967, 1968, 1970a, 1970b; Piaget and Inhelder, 1971) -- the sensori-motor structures. Logical (operational) structures are not generated from the figurative schemata, i.e., not from perception, memory, etc. Reciprocally, figurative structures do not derive from operative schemas but from the representations of past states of events derived from physical experience. And most importantly, figurative structures do not derive from each other, but have unique bases in sensori-motor schema. Imagery, for example, is a derivative of deferred sensori-motor imitation (Piaget, 1951, 1952; Piaget and Inhelder, 1971) and not perception.

The postulation of these quite distinct functional structures is one of the cornerstones of Piaget's theory of knowledge acquisition and cognitive development (cf: Furth, 1969). Both the source and function of the structures are theoretically distinct. Operative structures derive from abstraction from coordinated actions, figurative structures derive from sensori-motor and perceptual activity. Operative structures produce "logical" transformations (conservation of invariants) whereas figurative structures reproduce sensory-perceptual consequences of externalized (and, thus "environmental") configurations. The variant operative structures of the in intuitive, the concrete, and the formal levels form the discontinuous sequence of stages of cognitive development. On the other hand, figurative structures are static and dependent directly upon the data of experience (sensory-perceptual consequences of stimulation). Piaget makes the fundamental assumption that all knowledge acquisition activity is constructive, but the construction of figurative representations is quite a distinct process from the constructive activity at the operative level.

Logically, there are three possible realtions between the figurative and operational structures (Langer, 1969). First, they may be unrelated and, if so, as mentally segregated functional structures, do not set limits on the functioning and development of each other. Second, psychological phenomena might be reduced to one of the types of structures. Langer (1969) suggests that subjective idealists, perhaps, try to reduce psychological phenomena to assimilatory operations; and there are many theorists who try to reduce all mental phenomena to accommodatory figurations; while naive realists propose that all knowledge processes are figurative (e.g.; perception is knowledge; Michotte, 1963; Garner, 1962). Third, and the one proposed by Piaget, is that of partial communication between figurative and operative structures within the constraints of the assimilation and accommodation processes.

The relations, and the potential form of interaction, are schematically presented in Figure 1 below.

Figure 1
Relations of Two Invariant Processes
of Adaptation and Two Types of Cognitive Structures


Adaptation
Equilibration
Conflict
Assimilation
Accommodation
Organization
Operational
Figurative
D
C
A
B
exchange events

Langer (1969) has examined the organizational and developmental (i.e., transitional) impact of accommodatory figurations on assimilatory operations (B, Figure 1); i.e., how does the child mentally extract and/or represent empirical information about physical and social objects and the consequences of that empirical activity for the construction of logical concepts. Imitation of an observed event, comparison of one's predictions with the perceived outcome of a physical manipulation, comparison of an observation or appearance (i.e., immediate experience) with the way things have been constructed and externalized, represent different modes of introducing internal conflict and cognitive-structural change. Generally, his findings are confirmatory, but not definitive with respect to the Piagetian hypotheses. In any case, if the development of each type of functional structure has implications for, but not direct causal effects upon, the structure and development of the other, current paradigms for the study of learning mathematical concepts will require considerable modification. The work of the Geneva group mentioned earlier, concerning for example, memory (A, Figure 1) and Langer's (1969) analysis of the impact of accommodatory figurations (i.e., imitation, etc.) on assimilatory operations represents beginnings in this direction.

Analysis of learning, in the context of Piagetian theory, poses requirements for much more detailed empirical analysis than has been generally recognized. On the one hand, researchers attempting to assimilate Piaget to their own conceptual structures concentrate on experimental procedures whereby the subject is required only to remember event contingencies (e.g., response-reward associations or "a" follows "b," follows "c"). Such procedures certainly produce change in "behavior" (e.g., Gellman, 1969; Mehler and Bever, 1967; Bever and Mehler, 1968); however, failure on transfer tasks, and a lack of persistence of task solution over time indicates that a figurative process underlies the change in performance. On the other hand, the accommodators (i.e., those more favorable toward Piaget's theory) often fail to generate experimental paradigms that adequately differentiate between the figurative and operational knowledge (Wallach and Sprott, 1964) or assume that "external disparity" (appearance vs. "reality") is sufficient to establish disequilibrium (conflict) between logical necessity (derived from the operational structures) and perceptual pregnance (cf: Bruner, 1966). Situations designed to establish disparity between the child's predictive judgment of the outcome of a transformation and his observation of the actual outcome may, in fact, generate little or no cognitive conflict. A most parsimonious explanation of many "negative" findings in training studies is, certainly, that such disparity belongs to the experimenter's "reality" and is external to the child's own logical operational system.

Implications for Learning and Instruction

In some form or other, the goals of American educators have always been stated in terms of "optimizing" the intellectual, social, etc., development of individual children; a vague statement, obviously, and subject to a variety of interpretations. Whatever imperatives that goal implies, the educational and instructional processes must be based upon an understanding of the nature of psychological development of children. Whether we want to produce individuals who will strive to maintain the status quo; individuals who desire and accept change; people content to be technologists (i.e., skilled labor); or problem solvers; it is necessary to understand the basic processes of child development and the conditions that permit "quality control of the product" (if I may use current jargon).

The issue is important because science (i.e., theory and research) can only yield "what is" and not what "ought to be." We are fortunate, in one sense, that the science of psychology (and of pedagogy) is young and imperfect; the proposed models and methods for educating young children are no less imperfect and are influenced as strongly by current social thought and individual philosophical biases, as by an understanding of the laws of psychological development. Such a state of affairs, while producing wasted efforts, spurious claims, more rediscoveries than discoveries, etc., can at least provide time for the development of articulated sets of societal goals for education.

The best that can be hoped for, under the current conditions of our knowledge, is development of preliminary "models" for instruction. Such models can provide, at least, a schematic set of principles and guidelines for constructing a learning environment consistent with the admittedly inadequate theories and knowledge of psychological growth. However, we should try not to violate recent advances in theory and known laws of child development.

Piaget, until recently (1971b), declined to generalize his theory to specifics for educational practice. His theory of knowledge acquisition, however, has contributed to clarification and integration of a set of propositions about psychological development, many of which have a long history in child psychology and education. If we accept his theory of cognitive development, several deductions concerning the construction of "optimum" environments can be generated. A modest attempt in this direction has been made at the University of Georgia for the Follow Through Program (Smock, 1969). Though the basic propositions of the model are not inconsistent with Piaget's thinking about knowledge acquisition, the interpretation is that of the modeler. It is influenced, therefore, by numerous sources of bias, misunderstanding, distortions, etc., that are inevitable under conditions where abstract theoretical concepts are not represented in unequivocal abstract or logico-mathematical terms.

We start with the general proposition that the child is not a passive recipient of stimulation, nor can external reinforcement be considered a primary factor in learning and behavioral change. Further, the introduction of "mediation responses" (verbal or otherwise) is not able to account for the complexities of observed changes in behavioral organization during the course of psychological growth during childhood. Many psychological theorists have adopted, in one form or another, the idea that human organisms actively respond to their environment and that the patterning of these responses reflects a "plan" or "set of cognitive operations." In other words, the child "interprets" environmental input, and the interpretations are controlled by his capabilities for generating rule systems for coordinating and transforming the input to "match" a scheme, plan, or a mental operational structure. Analysis of the "rule systems" characterizing cognitive development, thinking, and learning, requires specifications of the properties of, and antecedent conditions for, selection and structuralization of the consequences of environmental events (mental representation/figurative knowledge) and of the mental actions (operative knowledge) necessary for coordination and transformation of those representations. The study of the development of rule systems defined as such is coincident, then, with the systematic investigation of the "inherent logic" of development of operational and figurative thought processes.

Intelligence, first of all, is considered no more, and no less, than biological adaptation; i.e., adaptation at any level of complexity reflects "intelligent" activity. "Knowledge" consists of two types of functional structures (figurative and operational) that give rise to invariants in organism-environment relations. These invariants are derived from abstractions from objects (physical experience) in the first case, and from coordinated actions (logico-mathematical experiences) in the second. Intelligence, then, refers to both types of cognitive learning and is defined in terms of functions (i.e., thinking reasoning) rather than content (i.e., words, verbal responses, associations, etc.). Analysis of conditions for cognitive learning and development must begin with the identification of components of behavioral organization (structure) that reflect particular coordinated action-modes of the child as he is confronted with changing intrinsic (maturation and prior cognitive acquisitions) and extrinsic (physical and sociolinguistic) factors.

Cognitive structures or systems of coordinated (mental) actions proceed through invariant stages of structural change with ontogenetic development. The successive differentiation and hierarchical integration of these cognitive structures permit the individual to cope with increasingly complex social and physical "realities." The process of cognitive development involves the changing characteristics of transformational rule systems (virtual and/or cognitive operations) characterizing the child's mode of adaptation. Neither the maturational structure of the organism nor the "teaching" structure of the environment is the sole source of reorganization; rather, it is the structure of the interaction (exchange events) between the child and the perceived environment that provide the basis for intellectual development.

Optimal conditions for structural organization and reorganization require: a) an optimal degree of discrepancy between perceived environmental/demand structures (i.e., perceptual activity, images, memories) and cognitive operational structures; and b) social-learning conditions that demand "spontaneous" or "constructive" activity by the child.

Several implications for the construction of theoretically appropriate learning environments are implied in these general principles. First, structural change depends upon experience but not in a way that traditional learning theorists conceive experience; i.e., learning interpreted as pairing of specific objects and responses, direct instructions, modeling, etc. Rather, the functional genetic view holds that the cognitive capacities determine the effectiveness of training. For example, ability to solve class inclusion problems implies that the child already has the requisite single and multiple classification operational system for classes (i.e., combination, reversibility, etc.) in addition to appropriate information selection, storage, and retrieval abilities. At the same time, while experience is necessary for developmental progress, and appropriate enrichment of the environment can accelerate such development, experience cannot change the sequence, structuring, or emergence of action modes in the process of developmental change. In other words, organization of experience is not provided solely by the environment nor solely by the structures intrinsic to the child.

Second, the structure of learning environment must be considered relative to two frames of reference; that is, in terms of the operational systems controlling the child's interpretation of "environmental" events and the content to be learned.

Operational systems are expressed behaviorally in the coordinated actions of the child confronted with changes in his physical and social world. For example, the mental operations of associativity, reversibility, etc., are inferred from the manner in which the child attempts to solve problems involving regular environmental contingencies (causality), understanding of spatial relations, arithmetic, etc. Substantive areas (e.g., science, mathematics) also must be analyzed in terms of their own logical sequence and/or commonalities with other content areas. Certain concepts in the physical sciences, language, and mathematics, for example, may have an inherent sequence and structure; thus, certain concepts may be necessary precursors to subsequent understanding of higher order concepts. Optimal educational conditions require, then, thorough understanding of the psychological-cognitive capacities of the child as well as the sequential structuring of concepts within a particular curriculum area.

Third, the striving for equilibration between assimilatory and accommodatory processes under both intrinsic or extrinsic pressures underlies the adaptive process. Optimum conditions for structural reorganization (learning in the broad sense) require disequilibration. This condition is met when there is an appropriate "mismatch" between the cognitive capacities of the child and the conceptual demand level of the learning task. Too little or too much "pressure" may result in over-assimilation or over-accommodation respectively, and not promote the desired developmental-cognitive changes.

Fourth, facilitation of learning requires analysis of two levels of cognitive functioning -- figurative and operative processes. The first (i.e., figurative thought) is most emphasized by those theorists (particularly behaviorists) recommending a direct tuition approach to instruction. The operational theory of intellectual development does not deny the value of "provoked" learning (i.e., through imitation, algorithms, etc.). Rather, it must be recognized that such learnings are considered limited because of lack of generalization or transfer to new situations and because the basic (i.e., operational) intellectual processes concerned with problem solving and reasoning are not much affected.

While there is some doubt that much acceleration of structural reorganization is possible through environmental enrichment, early childhood education should provide opportunities for utilization of relevant cognitive operational structures. Generalization of conceptual learning across content areas rather than the building of specific knowledge and skills (e.g., a large vocabulary) should be emphasized since the latter cannot directly accelerate operational system change and may, in fact, retard development of these "deeper" competence structures.

In any case, the nature and variety of the child's "exchanges" with the environment need to be considered in educational planning. The nature of the interaction refers to the relative emphasis on ontogenesis (self-directed) as contrasted to exogenesis (environmental or teacher-directed) structure of the learning environment. The functional genetic position can best be summarized in the old adage -- "You can lead a horse to water, but you can't make him drink - unless you feed him salt." Thus, the task of the teacher is to engineer an educational environment consisting of curriculum materials, social interactions, and directed activities that provide appropriate "salt" for each child. Sequentially structured curricula should be designed to provide an optimum degree of structure and conceptual level to permit an appropriate balance of assimilatory and accommodatory activity.

The variety of interaction (i.e., enrichment) refers to the types of structured curriculum contents relevant to the child's physical, social, and symbolic experiences. The interlocking nature of substantive curriculum areas makes it possible to provide a variety of experiences relevant to acquisition of the cognitive "products" that provide representation of the environment (memories, vocabulary, etc.) and, at the same time, to facilitate the development of coordinated rule systems associated with cognitive operational development. For example, analysis of the visual environment (attention or observational skills) as well as cognitive operational structures (e.g., conservation of are) can be emphasized in science, social studies, mathematics, art, etc.

The engineering of an educational or "learning environment" based on the preceding consideration necessarily involves the development of specifications of: 1) the child's cognitive developmental level; 2) the physical structures, including curriculum materials; and 3) the social or interpersonal structures. The organization of these "elements" should be such that equilibration, between different cognitive systems and/or between intrinsic functional structures and "environmental" structures, is achieved. Thus, sequentially structured sets of curriculum materials and of social interaction situations are necessary to provide the "pressure" necessary for learning (adaptation). A variety of specific learning environments needs to be available to maximize the probability of each child's finding activities that attract or "trap" him into interacting with the physical (e.g., science) and social (e.g., social studies) environment at both the behavioral and symbolic levels (e.g., language and mathematics) in creative and spontaneous ways (e.g., art, role playing, music). Finally, the physical and social environments should be arranged so that considerable freedom of movement, within the structure of a variety of contents, is possible; i.e., "a modified open-structure classroom." A careful balance between relatively high and low structured learning situations and between group and individual learning activities should be maintained.

The Mathemagenic Activities Program, a model developed in the context of enriching the educational environments of economically deprived children,[3] is based on three explicit principles derived from the considerations discussed above. Specifically, the MAP principles of change -- whether the target for change is the individual organism (child) or a complex social system (e.g., Local Education Authority) are based on the above assumptions concerning the role of experience in learning and development. First, the source of motivation to change is provided by a discrepancy (disequilibration) between different conceptual systems (ideas) and/or between previously acquired conceptual systems and environmental task demands. Thus, an appropriate mismatch (M) is necessary to generate exploratory activities and insure the individual has the prerequisite conceptual basis for learning higher order concepts.

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[3] Mathemagenic Activities Program: A Model for Early Childhood Education, prepared by C. D. Smock (A preliminary statement of the conceptual basis for the Mathemagenic Activities Program for the Follow Through Program appeared in Terminal Report, Research and Development Center for Early Educational Stimulation, University of Georgia, August, 1970).

Second, since coordinated actions (practical and mental) are the bases for knowledge acquisition, the learning environment must be structured so that specific task demands include appropriate practical, perceptual, and mental activity (A).

Third, the learning environment must include provisions for personal, i.e., self-regulatory, (P) constructions. Knowledge acquisition involves construction of invariants from properties of objects (physical experience) and from the child's actions on objects (logico-mathematical experiences). Optimal conditions for facilitating new "constructions" (concept learning) involve a balance between tasks that are highly structured (in which the child merely "copies" or imitates the correct solution) and tasks that permit the child to generalize and discover new applications of his concepts. Practically, self-regulation implies a variety of task options available to the child; the number of options may well vary with the nature of the task and many other factors. MAP proposes, however, that options -- in terms of level of task difficulty, mode of learning, and choice of activity -- are necessary ingredients of developmental change, whether the target be a child, a teacher, or an educational system.

The implied educational model requires significant changes in the teachers' role definition and teaching strategies (as well as tactics). The need for sensitivity to the child's capabilities, and the structuring of learning situations that promote self-regulated, "constructive" knowledge acquisition, together with thorough acquaintance with available technological aids, require an "educational engineer" in the best sense of that term.

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Radical Constructivism and Piaget's Concept of Knowledge

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Preface

Piaget's influence on theory, research, and practice in psychology and education is well known. What is not so well recognized is the fact that his notions about the nature of knowledge and knowledge acquisition constitute an intellectual revolution for the behavioral sciences that is comparable to that generated by Einstein's theory of relativity in physics. Relatively few American scientists or educators have recognized the full implication of Piaget's constructivistic position for theory building, for research methology and for application in the social sciences and education.

The MAP Follow Through program is based on principles of developmental change derived from Piaget's constructivistic theory of knowledge acquisition. The paper presented here is one of several MAP publications that attempt to disseminate our interpretation of Piaget's epistemology and its implications for educational theory and practice.

C. D. Smock

E. von Glasersfeld

Abstract

Unlike the majority of contemporary psychologists, Piaget has never dodged the epistemological question that the study of human behavior, and especially of cognitive behavior, inevitably raises. In fact, in almost everything he has written, he touches upon the problem of knowledge and how we acquire it. Nevertheless it is not easy to come to a clear understanding of Piaget's theory of knowledge. This paper attempts to show that his Genetic Epistemology entails a drastic break with the traditional conception of "knowledge" and I interpret this break from a "radical constructivist" point of view. I have called this constructivist approach "radical" because it embodies not only the view that cognition must be considered a process of subjective construction on the part of the experiencing organism, rather than a discovering of ontological reality, but also the belief that there can be no rational access to any world as it might be, prior to, and independent of our experience.

After a short exposition of the traditional view of cognition and the concept of interaction between organism and environment, the epistemological ideas that have recently been developed in the field of cybernetics are outlined. The emerging radical constructivist model is then applied to the Piagetian concept of knowledge and it is suggested that this interpretation, though rarely explicitly stated, is implicit in Piaget's Genetic Epistemology and is, indeed, necessary in order to make it an internally consistent theory of knowledge.

Radical Constructivism and Piaget's Concept of Knowledge (*)

The Traditional Approach to Cognition

In the main stream of philosophical tradition, ever since the pre-Socratics, the questions "What is knowledge?" and "How do we come to have it?" are inextricably tied to the concept of truth and of reality. Whether or not it is explicitly stated, "knowledge" is conceived as the knower's representation of things and events "in themselves" as they are supposed to exist in a "real" world, i.e., a world that is thought to "be" prior to and independent of the knower's cognizing activity. This was the general view before Kant, and, in spite of his well-founded reservations, it still is the general view among psychologists and the bulk of other scientists. The one significant exception are the great physicists who, during the first third of this century, demolished the naive realism of classical science and revolutionized the physicist's picture of the universe.

The traditional view of cognition, as Maturana (1970) says, begs the basic epistemological question, because it tacitly presupposes both a reality to be known and the knowing organism's possibility of discovering it. Inevitably, any view that conceives of "knowledge" as a representation, image, or replica of structures that are there prior to the knower's act of cognizing, runs into an unsolvable problem: How could the knower's representation ever be said to reflect, or correspond to, or approximate, reality, if the only access he can possibly have to it is his very own activity of "knowing"? To establish the "objective truth" or "validity" of such a representation, some form of non-cognitive comparison or match with that supposed independent "reality" would

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(*) This paper elaborates on ideas first expressed in 'Assimilation and Accommodation in the Framework of Piaget's Constructivist Epistemology', presented at the Third Southeastern S.R.C.D. Conference (Chapel Hill, N.C., April 1974) and in Epistemology and Education, C. D. Smock and E. von Glasersfeld (Eds.), Athens, Georgia: Mathemagenic Activities Program-Follow Through, 1974. The author gratefully acknowledges the many helpful comments and criticisms from graduate students and from his colleagues C. E. Smock, John Richards and Stuart Katz.

be indispensable. Such a comparison, however, is doubly impossible. First, because there can be no way of comparing what we have experienced to what we have not experienced. Second, because - whatever else it might mean - "to know" means to cut up experience into repeatable, recognizable segments (cf. Brown, 1969); and again, there can be no way of comparing these segments, and the structures we build out of them, to the un-cut and, therefore, unstructured whole of our experience.

Experience and Subjectivity

Many scientists - just as craftsmen, engineers, and technicians in general - can carry on with their jobs perfectly well, without ever asking a question concerning the relation between the perceiver and the perceived, or between the knower and what he comes to know. They can do so, as long as the items and events they are concerned with are all contained in their field of experience. In that case, items, and the relations between them, can be considered the "causes" of other events or states which then become their "effects". Both, the causes and the effects, will be accessible to experience, at least in principle. That is precisely what is required by theories of "verification" or falsification" as they have been advanced by logical empiricists[1] and by Karl Popper (1934/1965: 33). Inferences drawn from experience are comfirmed or disconfirmed by further experience. From an epistemological point of view, it is important to stress that confirmation may make an inference or hypothesis more plausible, or valid, or even more useful, in that it shows it to have held over a larger population of experiences. This may lead us to make some other inferences about how we structure our experience, but it cannot possibly justify inferences about the ontological status of the experienced items and the relations we have posited between them.

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[1] Cf. for instance Quine (1953/1963: 41): "The factual component of statements must, if we are empiricists, boil down to a range of confirmatory experiences."

If existential inferences are proposed, they are always, explicitly or implicitly, causal inferences. What we experience, they suggest, is to be considered the effect of something that "exists" prior to our experiencing it. The causal connection, in this case, is posited on the basis of "analogy" to what we do within our experience. In order to systematize experience and to make predictions about it, we are constantly inferring causal links between segments of it, which after Hume, we may call events A and events B. An existential inference, however, requires that we consider the elementary segments of our experience as D's, i.e., as effects caused by C's that are prior to our experience and, therefore, lie outside it. Clearly, this is not the kind of "generalization by analogy" that scientific methodology sanctions. Analogy can be invoked, if we believe that C is like A and D is like B (Kaplan, 1964: 107). In this particular case, we never have been, nor ever will be, in a position to say what an item of that supposed pre-existing reality is or is not like, before we have actually experienced it. And since, afterwards, all we can possibly know is what we have experienced, the generalization of the causal connection is wholly illegitimate. Thus, though we may intuitively believe in a structured ontological reality, to invest it with the power to "cause" our experience is no less an act of mythical faith than to invest it with the power to punish or reward us or to protect us from the things we happen to dislike.

Organism and Environment

When, however, a scientific investigator does ask the question concerning the relation between the perceiver and the perceived (or, more generally, the relation between the experiencer and what he experiences), he introduces an element that wholly changes the situation. If he observes an "organism", i.e., an item which he invests with the capability of perceiving and "behaving", he has already discriminated that item from the rest of his, the observer's, experiential field. Thus, he can meaningfully speak of the organism's "environment" when he wants to refer to all that part of his own experiential field that is not the organism. The relations he then sets up between the organism and its environment will still be relations between parts of his, the observer's, experiential field. Anything he then says, or might want to say, about the organism's experiential field must necessarily be in terms of (or derived from) his own observer's experiences. This does not mean that the organism's experiences must always be in every way the same as the observer's. They may be composed of different coordinations of elementary segments of experience and/or they may be made up of a smaller set of segments; but, as far as the observer is concerned, he cannot meaningfully include in his analysis or description of the organism elements that are not bits of his own experience.[2]

This limits the observer's explanation and understanding of an observed organism to what Maturana (1970) has called their "domain of interaction", a limitation partially foreshadowed in Klüver's (1965) doubts concerning "stimulus equivalence". Within the domain of interaction, however, the observer may speak of causes and effects, placing the causes in the observed organism's environment as "stimuli" and attaching the effects to the organism as "observable responses". By and large, that is what behaviorism has led psychologists to do, and insofar as they have tried to stick to the rigid program, they have striven to remain "technicians", i.e., to limit their explanations and interpretations of "behavior" to the linear cause-effect pattern of mechanics.

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[2] We may observe an organism and conclude that it is "color blind", i.e., that the elements in our own experience, which we call "color", are not part of the organism's experience. Or we may conclude that an observed organism directly experiences elements which to us are accessible only indirectly via transformation from one sensory mode to another by some instrument; e.g. some of the sounds auditorily perceived by bats or dogs. But we cannot conceive of elements of experience to which we cannot accede either directly or indirectly.

But since, in mechanics, there is no room for the transformation of energy into information, nor for inferential processes, their analyses of behavior could not come to terms with the very factors that make behavior essentially different from mechanical action and interaction. In virtually everything a psychologist may want to investigate, perception and/or cognition are involved, and the reason why we want to set these processes apart from the processes covered by mechanics is precisely their "informational" aspect. In cybernetic terms, that means that we want to specify relations between events, not as energy transformations and by means of energy equations, but semiotically as signs and their interpretation (Rosenblueth et al., 1943; Haldane, 1955). In other words, the process of coding, which is a prerequisite wherever we speak of information, inference, or cognition, involves the institution of a connection (semantic nexus) that is quite independent of the physical or mechanical characteristics of the items that are being connected.

The Interactionist View

The various "interactionist" theories start with the explicit assumption of two types of entity: organisms and environments. An environment impinges on everything in it and thus provides for "selection". Certain items will last, others will not. Living organisms are seen as a product of this principle. They have been "selected for" because, whatever it is we call "to live", it constitutes a specific way of dealing with certain aspects of the environment. What is alive is not only "adapted" as a category or species, but also has some capability of individual adaptation. In this view, then, perception and cognition are, on the one hand, capabilities that have resulted from adaptation by phylogenic selection and, on the other, they are instruments of ontogenic adaptation. If this is granted, we would seem to be justified in inferring that an adapted organism "reflects" his environment - at least in the sense that the organism's biological structure, being one that has manifestly survived, gives us some clues as to the structure of the organism's environment. Similarly, the results of perception and cognition, insofar as they can be shown to enhance an individual's adaptation, could be said to "correspond" in some sense to the structure of the environment. In short, with regard to the basic problem of epistemology, we would seem entitled to say that, though our knowledge of the existing world may be limited and somewhat distorted by our specific characteristics as organisms, we can acquire some knowledge of the real world.

There are two snags to this generalizing conclusion. The first we have dealt with above: What we conclude about organisms and their environments, when both are within our experiential field, can in no way be generalized to our own position, since our environment, i.e., an independently "existing" world, is not accessible to us except through our experiencing it. Hence, whatever we say about organisms we observe cannot serve as a logical analogy to ourselves qua organisms in an intuitively posited "real world" environment. It can be no more than a mythical metaphor.

It does not help either, to present the problem in crisp terms and, then, to say, as Attneave (1974) recently did, that its solution is the business of epistemologists and that he, the psychologist, prefers to go on believing that knowledge does reflect an "existing" reality. Such an act of faith is a rationally unfounded assumption and will sooner or later interfere with the investigator's logic of theory construction and, hence, also with his interpretation of "data".

The second snag is that the terms "selection" and "adaptation" are somewhat misleading. The theory of evolution from which they originate is essentially a cybernetic theory, in that it is based on "restraints" rather than on causation (Bateson, 1972: 399 ff.). "Selection" means no more than the elimination of the non-viable; and an "adapted" organism is merely one that comprises no feature, biological or behavioral, that would significantly increase the probability of the organism's elimination (prior to its having procreated others). Adaptation, therefore, can never be said to reflect the structure of the "real" world, because, even if it did, we could not possibly know it. Similarly, from maladaptation and extinction we can at best infer some organismic features that seem incompatible with the extinguished organism's environment as we see it. That is, we may gather indications as to what that environment does not allow. But a description in negative terms cannot be turned into a positive description, because the exclusion of some possibilities, in a field of infinite possibilities, does not make that infinity finite.

The trouble with the "interactionist" view is that, in fact, it presupposes much more than the two explicitly made assumptions. The generalized separation of organism and environment takes for granted an objectively existing space in which the two can be separated. Any assertion of action, by the environment on the organism or vice versa, takes for granted the objective existence of time in which these actions and their "selective" effects take place. Even if we are not prepared to follow Kant to all his conclusions, we cannot avoid admitting that he has successfully deprived time and space of ontological reality and turned them into characteristics of our particular way of experiencing (Anschauungsweisen). This is corroborated, on the basis of very different studies, by the physicist Schrödinger[3]: "The space-time continuum must not be assumed to be conceptually prior to what has been considered its contents; just as the corners of a triangle are not there, before the triangle." Hence, if space and time are prerequisites of "interaction", an interactionist view or theory can never legitimately prove the existence of an environment or of a "real" world, because it has a priori posited the spatio-temporal framework without which the separation of organism and environment, and their interaction, are meaningless.

The Contribution of Feedback and Control Theory

The birth of cybernetics as a discipline in its own right can be collocated with the publication of the paper by Rosenblueth, Wiener, and Bigelow (1943) which supplied a scientifically usable and, indeed, useful definition of "purpose". Future historians may explain why, for 30 years and more, the behavioral sciences disregarded that formidable conceptual conquest, while the builders of mechanical and electronic control devices, who adopted it at once, proceeded from one empirical success to another.

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[3] Schrödinger (1956: 24): "Das Raum-Zeit-Kontinuum darf nicht als begrifflich früher angesehen werden als das, was bisher sein Inhalt genannt wurde; so wenig wie etwa die Ecken eines Dreiecks vor dem Dreieck da sind."

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In the context of the present paper, the relevant aspect of feedback theory is the epistemological one. The heart of the theory is the concept of negative feedback. A system is considered to incorporate a control device if it serves a homeostatic function in that it is capable (within certain practical limits) of keeping a sensory "input" signal quantitatively close to a pre-established reference value. Thus it must comprise a comparator (the material implementation of this is irrelevant) that "compares" the sensory signals to the reference value. If this comparison yields a difference (negative feedback), this difference generates an error signal which, in turn, activates an "output" function, i.e., triggers an activity apt to result in the modification of the sensory signal (adequation to the reference value).

Such a feedback control system is a highly successful and versatile model for the analysis of the behavior of living organisms (Powers, 1973). It differs from the stimulus-response model in that it posits a circular rather than a linear cause-effect chain. While S-R theory attempts to link an organism's responses to specific stimuli, in the feedback model it is, on the one hand, the difference between a sensed quantity and the reference value that "causes" an active response and, on the other, it is this response that "causes" a change in the sensed quantity and, thus, the elimination of the error signal.

For an analysis of cognitive processes this development of feedback theory has several important implications. If behavior can be seen as directed towards the adequation of sensory signals or percepts rather than aimed at the modification of an "outside" environment, the process of learning and the structures (motoric or cognitive) that result from it, are no longer dependent on any match with an "outside" world, but only on the experientially "correct" association of effector functions with subsequent changes in sensory signals. In other words, the logically impossible match with an objectively existing ontological reality is no longer necessary and the organism can now be seen and described as operating wholly inside his own field of experience, i.e., operating towards internal equilibration.

In this model, the concept of "knowledge" is in one important respect radically different from the concept of knowledge in traditional philosophy, science, and common sense thinking. It explicitly excludes any idea of "reflection of" or "correspondence to" an independent objective reality supposed to "be" or "exist" in itself and awaiting a perceiving and/or cognizing organism to discover it. Instead, "knowledge" now designates any structure, i.e., any coordination or association of individually recurring elements of experience, which an organism uses in the ordering and systematization of his experience. Genetic epistemologists should have no difficulty in agreeing with this. Piaget has said it often enough: Knowledge is whatever an organism holds invariant in the changing flow of experience.

The Concept of Knowledge in Genetic Epistemology

In Piaget's view, a living organism begins its intellectual or cognitive career with some genetically determined "reflexes". A reflex, by the observer's definition, is made up of a stimulus and a response, both of which recur contiguously and in that sequence. From the regularity of their recurrence, the observer infers a "fixed" connection between the two. But what would it look like from the organism's point of view? The stimulus, simply enough, would have to be a sensory signal or, more accurately, a constellation or pattern of sensory signals. The response, analogously, would be a signal or constellation of signals going to an effector function, i.e., a kind of signal which even the most introspectively inclined organism does not claim to be aware of. Again Powers (1973) has supplied an extremely insightful formulation: to the cognizing system everything is input. This means that an acting system can register, record, or define its activities exclusively in terms of signals from proprioceptive or kinesthetic sources and their experiential coordination (Hume's contiguity) to subsequent changes of sensory signals. Important though this point is, it is not what interests us here. The point I want to stress is that, while the terms "reflex" or "fixed action pattern" focus attention on the coordinated or patterned character of the activity, they do not make equally clear that the stimulus that triggers the activity, usually, also has the character of a coordination or pattern. Thus, it would perhaps be better to define a reflex as consisting of a fixed connection between a specific perceptual pattern and a specific motor pattern (i.e., a scheme).

This detail, I believe, is essential if we want to understand the Piagetian terms of "assimilation" and "accommodation" in an epistemologically consistent way.

> Any new acquisition consists of assimilating an object or a situation to a previous scheme by thus enlarging it. . . . the thumb stimulus releases the sucking response only if it assumes a significance as a function of the scheme of this response, that is, if it is assimilated as a sucking object. (Piaget, 1974: 69-70)

Assimilation of the thumb as a "sucking object" means no more and no less than this: there is a pattern of sensory signals which, owing to a genetically fixed connection, triggers the sucking activity in the infant; from the observer's point of view, this pattern, while first elicited by the nipple of the bottle or the mother, is now elicited by the infant's thumb as well. At that level, and with reference to the scheme we call "sucking reflex", there is, for the infant, no difference between nipple and thumb. A difference can be made only when some other sensory signals, for instance the taste of milk, are coordinated with the original sensory pattern - and such a novel coordination, resulting in a more comprehensive recurrent pattern of sensory signals, would constitute an accommodation. If we ask how this accommodation comes about, we can at once say, it is induced by the very fact that, when the sucking response is triggered by the thumb, it does not lead to preception of the taste of milk, while if it is triggered by the nipple, it does; i.e., it creates a mismatch or "disturbance". The disturbance can be eliminated either by cutting the link between the "thumb" percept and the sucking scheme, or by differentiating two sucking schemes: one that leads to perceiving the taste of milk, and one that does not. In either case, what has to occur is another accommodation. Accommodation, thus, can be seen as the result of discrepancies or disturbances that arise from acts of assimilation when the assimilated item in some way modifies the outcome of the scheme into which it was incorporated. In the above example, the perceptual "thumb" pattern will eventually be coordinated with other perceptual signal patterns from tactual, visual, and proprioceptive sources, and will thus come to constitute a "permanent object", i.e., an item that is accessible and constructible in more than one sensory mode.

Though the observer of all this can meaningfully speak of "inside" and "outside" with regard to the observed organism, because he can separate the two in his own field of experience, the organism itself "assimilates" and "accommodates" and, quite generally, operates and acts exclusively on and with the signals that constitute its experience. The organism has no way of matching these signals or coordination patterns of signals with any outside reality, nor does he need anything beyond the sufficient availability of signals in order to continue with his construction.

"At eighteen months to two years this 'sensorimotor assimilation' of the immediate external world effects a miniature Copernican revolution" in the infant, and, as Piaget (1969: 9) goes on to say:

> at the termination of this period, i.e., when language and thought begin, he (the infant) is for all practical purposes but one element or entity among others in a universe that he has gradually constructed himself, and which hereafter he will experience as external to himself.

Piaget's meaning seems perfectly clear in this passage. The "Copernican revolution" that leads to the child's externalization of his sensorimotor constructs, which include not only "permanent objects" but also the sensorimotor basis for the construction of the concepts of space, time, and causation, is Piaget's key to the adult's belief in an external "prefigured" reality. This act of externalization has to be constantly kept in mind if a logically consistent epistemology is to be gleaned from the Piagetian theory of cognitive development. Whenever we read, for instance, that the organism's cognitive activity is "to assimilate the external world into the structures that have already been constructed, and secondly to readjust these structures as a function of subtle transformations, i.e., to accommodate them to external objects," (1967:8) we must remain aware of the fact that this "external world" and the "external objects" it comprises are the sensorimotor constructs which we ourselves have externalized when we underwent the Copernican revolution at the age of eighteen months.

Conclusion

If my interpretation is not altogether mistaken, Piaget's concept of knowledge is one that in no sense involves the idea of getting to know an ontological reality that would have to be imagined as a prefabricated, fully structured world, existing by itself and waiting to be "discovered" by a cognizing organism. The concept of knowledge he has developed in his Genetic

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Epistemology seems wholly compatible and congruous with what a cyberneticist would call "knowledge" in a control system. In the feedback model it is the structures or coordinations of signals - neuronal events, if you will - which the system has come to coordinate recurrently in its effort to eliminate "disturbances", i.e., negative feedback that creates error signals because it is different from the reference value. For Piaget as well as for the cyberneticist the structures or coordinations may consist of sensory signals in perceptual patterns, of effector signals in motor patterns, or of both in complex action patterns or schemata. Assimilation and accommodation, thus, take place when inconsistencies or contradictions arise through the interaction of different levels of constructs; for instance, externalized objects and specific action patterns. This interaction of cognitive constructs is, of course, not at all the "interactionist's" interaction because it does not involve a "real" outside world. Though the observer of an organism can place that organism into an "environment", he cannot derive ontological conclusions from this way of organizing his experience, because both, the organism and its environment, are parts of his own experiential field and this can contain no more than he himself has constructed and externalized.

In another paper (von Glasersfeld, 1974), using a cybernetic metaphor, I have said that from the experiencer's point of view, ontological reality is like a "black box",in that he has no way of discovering what "is" and how it might be structured. Here I should like to stress once more that radical constructivism does not deny the existence of a world, but it does deny the possibility rationally to describe such a "real" world. The cognizing organism would have no way of determining or deciding whether or not his constructs in any sense reflect the structure of a "real"

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world, even if he could come up with structures that are not dependent on his concepts of space and time. The fact that our perceptual and conceptual structures are repeatable and seem at least to some extent dependent upon the availability of elementary segments of experience (out of which they are composed) in no way justifies the assumption that our structures reflect, or correspond to, "existing" structures; it merely shows that these structures have a relative degree of viability given the way in which we happen to have segmented our experience. If I make three dots on a blackboard in front of you, I am creating an opportunity within our domain of interaction, for you to experience three "bits" which in this context, we may call "elementary segments of experience." You are free to use these three bits in order to construct what we call a "triangle", i.e., to connect them by performing a set of specific operations. But you are equally free, if you happen to be in a "poetic" mood, to construct out of them a horizon and the morning star. Neither of these constructions can be said to "exist" on the blackboard. Indeed, the three dots could be coordinated into innumerable other structures, and each one of them would be determined by the operations you, the experiencers, perform, and only in a trivial sense by the three dots which, in this metaphorical example, constitute the raw material.

More complex and better examples are found in the history of physics. It is there that radical constructivism fully agrees with Kuhn's idea of paradigm shifts (Kuhn, 1970), interpreting paradigm shift as a "change in the habitual way of constructing". The more often a construct is repeated and the greater the number of larger structures in which it is involved, the more indispensable it becomes and the more "given" or "objective" it seems. If this is so on the level of the natural sciences, it is a thousand times more so on the level on which we construct and externalize our every-day environments.

Finally, then, it has to be stressed that radical constructivism, like any other theory in science or common sense living, is a product of inductive inference. It does not purport to supply a model of the "world of being"; it merely hopes to develop a model for the rational organization of our experience that may make it a little easier (than do traditional models) to sort out what we can know and what must remain the unfathomable mystery of the world in which we find ourselves living.

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A Constructivist Model for Instruction[1, 2]

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June, 1976

> Science is not just a collection of laws, a catalogue of unrelated facts. It is a creation of the human mind, with its freely invented ideas and concepts . . . . The only justification for our mental structures is whether and in what way our theories form . . . a link with the world of sense impression.
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> (Einstein)

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Preface

The Mathemagenic Activities Program is a model for early childhood education, sponsored by the National Follow Through Program, based primarily on Piaget's theory of cognitive development. The emphasis on structure of cognitive operational systems (logico-mathematical experiences) as the basis for understanding intellectual development provides mathematics educators with a unique opportunity for potential contributions to child development. This paper explores some of the implications of Piaget's theory for mathematics learning and teaching.

C. D. Smock, Director

INTRODUCTION

The fruitfulness of mathematics education research on problems and issues posed by Piaget depends on the collaboration of individuals in several disciplines. Certainly, many problems of concern to mathematics educators are common to those of developmental psychologists - but many are not. At the same time, the identification of psychological issues critical to the mathematics education researcher will determine the degree of progress to be expected from the current work in your field. These meetings provide a forum that will help isolate the problems of common interest to the two disciplines.

The influence of Piaget's work is so pervasive that one hardly knows where to begin. But, why not at the beginning? That is, Piaget's primary concern has been epistemology in the study of the nature of knowledge and knowledge acquisition. If we wish to understand Piaget's theory of cognitive development, and to conduct empirical investigation relevant to the implication of that theory, consideration of his epistemological foundations are essential. Essential, firstly because that foundation influences his theoretical constructions relevant to development of concepts of space, time, causality, etc. Essential, secondly because the setting of observational conditions to test that theory must meet certain methodological imperatives if the findings are to provide relevant information and not merely "noise".

We need also to begin with Piaget's own writing and research findings. In doing so, we must go beyond a surface critical analysis (i.e., beyond "obvious" weakness in design, number of subjects, etc.) to the structuring and organization of the research as examples of his epistemology and theory in action. I am not proposing that we not be critical - rather that we read to be sure that we do not engage in criticisms that represent the assimilative tendencies of the critic rather than contradictions inherent to the theory itself.

Everybody "knows" what is wrong with education, in fact the definitions of the problems by professionals are the same definitions of "everyman." And, in most cases, the solutions proffered by professionals are not only the traditional ones but often more trivial than those of the "amateur." Few have the courage to face two fundamental questions involved: 1) what is the purpose of the institution of public education, and 2) what is the nature of knowledge and knowledge acquisition? Failure to deal with the first leads to tinkering with trivial elements of a complex societal institution that should be a mechanism for guiding change (whether of the child or society) while unwillingness to confront the second produces, at best, temporary excitement about "innovations". There is hope, however; a few scholars are beginning to view the school as an important element in cultural change (Sarason, 1971; Smock, Graham, Silverman & Huberty, 1975). This paper, I hope, reflects an appropriate interpretation of the implications of one's view of the nature of knowledge acquisition for instruction of young children in mathematics.

Currently, the names Piaget and/or laboratory, especially mathematics laboratory, are sure to attract interest. While "laboratories" are gaining in fad appeal, meaning for such an instructional technique has remained loosely and ambiguously defined. Teachers who attempt to use such a teaching strategy typically have been forced to rely on a set of "rule of thumb" slogans such as: "concrete understanding before abstract"; "intuitive understanding before formalization"; "activities, then pictures, then symbols"; "discovery rather than reception methods"; et cetera. Unfortunately, such slogans refer to distinct instructional variables which often specify contradictory approaches to teaching if their range of appropriateness is not qualified and coordinated by at least an embryonic theory of instruction.

Educators appear to have little interest in theory construction despite the current interest in "model" building. Theory, too often, is used to justify instructional biases and the "laboratory" and/or "open-classroom" is considered only an instructional device, whereas theory construction is necessary to clarify the roles that different classroom structures have in educational research and practice. Also, a classroom laboratory should provide a context for research on problems relevant to specific aspects of the instructional process (variation of teaching strategies and techniques) and for discovering those psychological conditions critical for development of thinking in children.

However, in my opinion, a remark by Chesterton is quite appropriate: "It is not that they can't see the solution -- they can't see the problem." We have yet to identify the fundamental dimensions of educational and instructional problems facing the teacher. Key psychological principles for mathematics instruction and construction of a theory of instruction can be realized only after this first step has been achieved. A body of knowledge now exists in developmental-cognitive psychology that should have considerable utility for contributing to more refined theories of, and strategies for, instruction; i.e., for creating better school learning environments. An approach to the immediate task is to search for suggestions from developmental psychology that pose relevant problems for developing theories of instruction.

There is no paucity of choices of psychological "models" available from which to start the search. Berlyne, Bandura, Bruner, Scandura, Skinner, Suppes -- and of course Piaget. Each has proposed a set of ideas relevant to a theory of instruction requiring theoretical and empirical study. The selection of any one model, brings with it many hidden presuppositions and is determined in no little part by one's own preconceived notions regarding human development, learning and education. As a group, these models represent a virtual wonderland of exciting ideas. Each of us can understand Alice's dilemma better as we explore their fantasies (Suppes, 1972).

A theory of instruction must begin with an adequate theory of cognitive development and learning. No longer can we accept that statement as "obvious" -- and go about the business of generating a multitude of methods based on unorganized intuitive rules based on inadequate knowledge of the process of cognitive development in children. All educators need to return to the beginning and ask, not "how do we teach?", but rather "how do children learn?"

Modern developmental psychology provides a necessary, but not sufficient, body of know-how for identifying some of the fundamental issues, constraints, and facts associated with the process of generating a theory of learning and instruction. But, to imply and act as if psychology had become relevant to mathematics learning only AP (i.e., after Piaget), misses a fundamental point about the relation of the science of psychology to the science of education. It distorts the history of both. Piaget's theory of cognitive development should not be abandoned without clear understanding of why. The historical pattern in education and psychology seems to be one of enthusiastic adherence to a relatively novel theory -- with disappointment and rejection following close behind. The absence of serious controversial issues underlying much of the current research in cognitive development increases my concern that much which is valuable in Piaget's theory may be lost.

Natural Genetic Epistemology and Cognitive Development

Many psychologists, including myself, consider Piaget's clarification of the necessary bases of theory construction as important as his cognitive developmental theory per se. Implicitly and explicitly, Piaget was greatly influenced by advances in theoretical physics (Bridgman, 1927) during the 1920's and 30's. The fundamental aspects of relativity theory that cannot be ignored in psychological theorizing are that (1) conceptual judgments are always relative to the position of the observer and that (2) analysis of knowledge acquisition requires a description of its operational basis; i.e., the mental operations of the individual associated with the construction and maintenance of consistentpatterns (structure) of his continually transforming relations with his physical and social environments. Thus Piaget is unique in that his emphasis on a constructivistic theory of knowledge (Piaget, 1968, 1971a) is indissoluble from his interpretation of operationism. "Reality" is constructed, not imminent in mind, man or stimulus, and applies to the child and, contrary to some interpretations (e.g., Stevens, 1935), theorist alike.

The form of epistemology typical of American psychologists (cf: Mischel, 1971) has been naive realism. That orientation has been quite useful. Our epistemological preconceptions, whatever they may be, are part of our theoretical picture of the child. Kessen (1966) states the issue clearly: "The child who is confronted by a stable reality that can be described adequately in the language of contemporary physics, is child very different from the one who is seen facing phenomenal disorder from which he must construct a coherent view of reality" (pp. 58-59).

Analysis of cognitive learning and development is always "biased" by the fact of a context of preconceived ideas of reality (i.e., western culture) and a particular set of concepts or theory and selected observations. Piaget's approach to the analysis of the development of children's conception of space provides us an excellent example (Piaget and Inhelder, 1956). The specification of a conception of space toward which the child will most likely develop, i.e., that conception held by most adults, is the critical first step. Observations and interpretations of the child's behavior are organized around the specifications inherent in that "endpoint" of development. This is not an example of bad science or of inappropriate procedures but rather illustrates that conclusions about the child's cognitive structure is often more a function of the construction of reality imposed on him by the scientists rather than of the reliability and generality of "objective" observations. Whether we are engaged in instructional practice, research, or theory building, there is, for each of us, a set of guiding propositions that constitutes a theory of learning and development. These "fantasies" or "freely invented ideas and concepts" provide a particular coherent view of the developing child and of the presumed determinants of learning.

Piaget's constructivistic epistemology, and his biological background, predisposed him toward an operational and structural analysis of the knowledge acquisition process (Piaget, 1967; 1968; 1970a; 1970b; 1971a). The essentials of his position require only brief review there. Knowledge is defined as invariance under transformation (a most familiar concept to mathematicians). The construction of invariances in organism-environment relations takes place through the operation of two complimentary biological adaptation processes, both of which are under the control of the internal self-regulating mechanism of equilibration.

One of the two processes (assimilation) concerns the application of cognitive operational systems (structures) to the organization of sensory data.

New data or events are incorporated into existing structures through both on-going physical and mental activity. Such events and the products of new experience can be incorporated into the cognitive structure only to the extent they are consistent with existing functional structures. Accommodation is the complimentary process whereby adaptation occurs by integration of existing structures with other functional structures and/or by differentiation of new structures under confrontation with new experience.

Activities such as play, practical or symbolic, represent assimilative activity; whereas memory, in the sense of invoking past experience, and imitation are accommodative since only prior formed structures are transformed for a new use or application. Assimilation is an active constructive process by which the data from experience are transformed and integrated with an already generalized cognitive structure. Accommodative activity, on the other hand, is the process whereby modified existing structures or novel structures are brought to bear on newly assimilated sensory data.

Too often instructional theory and practice have emphasized assimilation (i.e., "play")or accommodation (i.e., imitation) activity and neglected the role of equilibration of these complementary processes for cognitive learning. However, generalization of Piaget's ideas to instructional theory and practice is not simple and straightforward. The special meanings attributed to "logic", the role of the equilibration mechanism in constructing experiential data, and the distinction between operative and figurative thought are critical to such an endeavor.

Operational Structures and "Logic"

Piaget recently (1970b) elaborated his position that all human beings possess the same biological structures and functions that, in "exchange" with the common features of the nature world, generate mental (operational) structures and functions characteristic of each stage of development. Logical thought, in the Piagetian sense, is univeral and of fundamental importance to an understanding of development and learning. But, whereas Chomsky maintains the human mind is "programmed" at birth with cognitive structures (i.e., mental representation of a universal grammar), Piaget accounts for the universality and stability of structures across cultures (Piaget, 1967; Goodnow, 1962; Goodnow and Bethon, 1966; Greenfield, 1966; Maccoby and Modiano, 1966) in terms of the self-regulation mechanism of equilibration. Thus, Piaget proposes that the mind at any point in development is the unfinished product of continual self-construction (1971b). Logical processes are generative and not fixed. Structures are not performed, but are self-regulatory, transformational systems with the functional factors in that construction being the processes of assimilation and accommodation.

Intelligence, the basis of knowledge acquisition, has two aspects: adaptation with the complimentary process of assimilation-accommodation under the self-regulatory mechanism of equilibration and organization consisting of sets of mental operations that form the basis for maintaining invariance under transformation (i.e., knowledge). It follows from these considerations that there is an inherent logic to development. Operational systems consist of elements and laws of combination of those elements that form a "logical" closed system. These mental structures are observable in the actions of the organism in its environment. They are describable in terms of formal or logico-mathematical structures. Genetic psychological analyses of these structures are a necessary prerequisite to an understanding of thought processes since there is "no structure without genesis, no genesis without structure."

During the sensori-motor period of development, action structures of the individual are revealed in "practical" groups, i.e., the coordinated actions of the individual (Forman, 1973). During the pre-operational period, the child constructs representations called figurative structures which do not have the operational property of reversibility. Piaget was able to identify operational structures with mathematical system properties in children between ages five and seven. The discovery of a resemblance between the structure of the mental action system (reasoning or thought) and mathematical structures (i.e., mathematical groups and lattices) had a profound effect on Piaget's thinking. Thought, it would appear, has the same or similar properties, as mathematical group structures and both are governed by the same internal logic.[2]

The basic structuralistic approach of Piaget involves finding or creating logico-mathematical systems that describe the thought processes of an individual. Mathematical group and lattice theory are algebraic systems (Piaget, 1957a, 1957b; Flavell, 1963) which might well describe operational thought. Piaget has tried to take maximum advantage of this approach. To review the fundamentals: a mathematical group is a system consisting of a set of elements, together with an operation (law of combination), which yields the following system properties: (1) when applied to the elements of the set, the combinatorial operation will yield only elements of the set; no elements external to the set can be produced; (2) each set contains a neutral (or identity) element that, when combined with another element of the set, yields the same element set; (3) each element of the set has an inverse belonging to the set which, incombination with any other element, yields the neutral or identity element; (4) the combinatory operation is associative; i.e., $[(n + m) + p = n + (m + p)]$. Piaget found it necessary to generate a "grouping" model with additional properties (i.e., both group and lattice properties) to describe the concrete operational structures. Most importantly, the properties of these psychological groupings are not derived from the properties of things, but from patterns in relation to things. Thus, the elements of psychological groups are themselves transformations that characterize the individual's operations as he acts upon sense data.

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[2] Piaget never has tried to find a mathematical "logical model" to "fit" the observed facts of behavior; rather the mathematical aspect of Piaget's theory is unique in that he assumes, somewhat a reminescent of Boole's "laws of thought," an identity between the inherent logic of thought processes and certain mathematical systems that have become formalized and "externalized," through inductive reasoning.

The revelations emerging from relativity theory require a constructivistic position with respect to the nature of knowledge. Understanding of knowledge acquisition requires a description and characterization of the mental operational systems applied to the data of experience. Piaget's emphasis on structural analysis thus is in terms of three traditions in modern intellectual thought: 1) the epistemological implications emerging from relativity theory; 2) the biologists' emphasis on development as the formation, differentiation and hierarchical integration of functional action structures, and 3) the mathematicians' emphasis on formalized systems that permit description of these structures. The task of the developmental psychologist is to describe the nature of action structures of the child at each point in development and, as much as possible, to formalize those descriptions in terms of logico-mathematical terms.

The classical "conservation" tasks, if administered appropriately, form one basis for generating observation of coordinated actions that appear to reflect such mental operational structures. The available evidence appears to support the hypothesis that such operational (mental) structures "exist" both in terms of replicability of hypothesized developmental trends and in appropriately designed training studies (cf: Beilin, 1971a). At the same time, neither psychological nor educational researchers have yet devoted sufficient attention to the problems of the validity (i.e., internal consistency) of the grouping structures (Clarey, 1971; Green,Ford, & Flamer, 1971) nor to the role of such structures in learning (Berlyne, 1965; Bruce, 1971; Inhelder & Sinclari, 1969), beyond these few studies.

Only recently have educators, especially mathematics educators, become interested in Piaget's views of fundamental logico-mathematical relations, such as his ideas about the logical properties of number and space. Beilin (1971b) points out that philosophers of science generally have emphasized the desirability of isolating philosophical and logical issues from psychological matters. Psychologists, mathematicians and logicians generally have maintained this position with respect to Piaget. However, a significant part of his psychological theory has mathematical and logical content which cannot be ignored by either psychological (Leskow and Smock, 1970; Alonzo, 1970) or mathematics-learning researchers. Mathematics education researchers rightly should be directed to the analysis of the logical and mathematical validity of Piaget's system *and* to the *correspondence* between the characteristics of the psycho-logic systems and those logic structures derived from purely mathematical analysis. Recent work from Steffe's laboratory (e.g., Kidder, 1973; Johnson, 1971; Johnson, 1972) represents an excellent beginning in this direction.

Role of Experience and Equilibration

Experience is not the only critical factor in development according to Piaget. Merely being exposed to particular environmental situations is conducive neither to cognitive activity nor to developmental change. Children may or may not make discoveries in the course of play. Watching a laboratory experiment or conducting one may or may not help a child acquire a particular concept. Equilibration is the central factor in structural change whether that refers to "stage" or to concept learning. Equilibration is the process of the adaptational structure "controlling" itself (intrinsic regulation); balancing assimilatory and accommodatory processes, compensating for external and internal disturbances (internal or external to a particular structure) and making possible the development of more complex, hierarchically integrated operational structures. Rhythms, regulations, and operations are the three essential procedures of the self-regulation and self-conservation of structures. Anyone is free to see in this the "real" composition of structures, or to invert the order by considering the operative mechanisms the source of origin. In any case, it is necessary to distinguish two levels of regulation. One level of the regulation remains internal to the already formed or nearly completed structure and, thus, contributes a state of leading to equilibrium. On the other level, such regulation participates in building up and integrating new structures.

Disequilibrium occurs as the child assimilates data from immediate experience into existing mental structures. As cognitive structures change to accommodate to the new experiential data, equilibrium is restored. The equilibration process is one of auto-regulation -- both of the transformations of data based on existing cognitive structures and of changes through accommodation. Thus, the child must be exposed to situations that are likely to "engage" the functional structures. He must be involved in a personal striving to understand or "accept" the task as a "problem".

A basic question for instructional theory and practice is: What are the processes and conditions that motivate and insure engagement or acceptance of the problem task by the child? The source of "interest" that promotes striving for problem solution is contingent on assimilative-accommodative activity but the specifics remain unclarified in Piaget's theory (cf: Mischel, 1971). Within a structuralist framework -- if a structures exists, it must function -- multiple cognitive structures provide a dimension of openness that make probable continual sources of disequilibrium from interaction of the internal operational and/or figurative structures activated, as well as by exchanges involving novel experience. And, despite lack of specifications, Piaget is quite explicit on his general position: "It is not necessary for us to have recourse to separate factors of motivation in order to explain learning; not because they don't intervene . . . but because they are included from the start in the concept of assimilation . . . to say that the subject is interested in a certain result or object thus means that he assimilates it or anticipates an assimilation and to say that he needs it means that he possesses schemas requiring its utilization" (Piaget, 1959, p. 86). In passing, it might be noted that natural or life-like contexts seem to provide excellent situations for promoting cognitive change. Unfortunately, too little empirical investigation has been oriented to questions of the natural environmental determinants of curiosity of children at various stages of development and with different experiential backgrounds. What do children recognize as problematic? What kinds of incongruities are sufficient to motivate change in concepts and/or beliefs?

Cognitive conflict, or the awareness of a momentary disequilibrium, generates a need to establish equilibrium between the existing schemas and/or novel information. This condition is the motivation for cognitive activities. Both application of an existing schema, and the elaboration of new ones in the course of development, stem simply from the overriding need to make "sense" of present problems by fitting them coherently into schemas "learned" in the course of solving prior problems.

The notion that disturbances introduced into the child's systems of prior schemas lead to the adoption of a strategy for information processing is the fundamental difference between the equilibration and associationistic theories of learning (Piaget, 1957b). For associationistic theories of learning, "what is learned" depends on what is given from the outside (copy theory) and the motive that facilitates learning is an inner state of some sort or other. Equilibration theory holds that learning is subservient to development; what is learned depends on what the learner can take from the given by means of the cognitive structures available to him. Further, cognitive disequilibrium is the functional need that motivates learning. Questions or felt lacunae arising from attempts to apply schemas to a "given" situation are disequilibrium. The child will take interest in what generates cognitive conflict or in what is conceived as an anomaly. If the task demands are so novel as to be unassimilable or so obvious as to require no mental work, the child will not be motivated.

After the period of sensori-motor development, equilibration becomes a process of compensating for "virtual" rather than actual disturbances. At the operational level, intrusions "can be imagined and anticipated by the subject

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in the form of the direct operations of the system -- the compensatory activities also will consist of imagining and anticipating the transformations to an inverse sense" (Piaget, 1957b, p.93).* Further, there need be no external intrusions in order for the equilibration process to be activated. For example, the acquisition of conservation concepts is, in Piaget's view, "not supported by anything from the point of view of possible measurement or perception -- it is enforced by logical structuring much more than by experience" (Piaget, 1957b, p. 103).* It is the "internal factors of coherence -- the deductive activity of the subject himself" that is primary. Equilibration is a response to internal conflict between conceptual schemas rather than a direct response to the character of environmental structure factors. Bridgman (1936) expresses the same idea when he states "we do not experience things; things are a construction of ours the function of which is to emphasize the resemblance between aspects of our present immediate experience and aspects of our past experience. . . " (pp. 18).

Operative and Figurative Thought

A considerable amount of confusion concerning Piagetian theory and its implication for both research and instructional practice derives from a failure to consider the figurative and operative aspect of intellectual functioning. In general psychological terms, the distinction is between the selection, storage and retrieval versus the coordination and transformation of information (Inhelder, et al., 1967). More specifically, the development of any sequence of psychological stages, a la Piaget, consists of an interactive process of equilibrating functional structures of the organism with sensory event-structures of the perceived environment.

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*Translation by E. von Glasersfeld

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Piaget (1970a) analyzes "experience" into the two components of "physical" and "logico-mathematical". This distinction between a physical and logico mathematical experience is essential to the understanding of the growth of knowledge. Knowledge based on physical experience alone is knowledge of static states of affairs; if a child is wrong, it is easy to demonstrate that to him. While knowledge emerging from logico mathematical experience is knowledge of transformation of states and quite another matter. If a child is wrong in the first case, it is easy to demonstrate his error, but in the second it is difficult, if not impossible, to demonstrate convincingly, or even to get the child to accept verbal explanation of the "correct" answer. For example, if a child fails to align the two endpoints when comparing length of sticks, it is easy to correct the mistake. If, however, he fails to display transitive reasoning in a task, one or two examples is not likely to "teach" him the concept.

Physical experiences provide for the construction of the invariants relevant to the properties of states of objects (figurative processes) through exchange with objects involving sensory mechanism. For example, one may touch something and it is hard, cold, hot, soft, supple, et cetera. Or one may see something -- an object is red, a diamond cutting glass, the shape of a banana, et cetera. The course of logico-mathematical experience is assumed to be abstractions (operative knowledge) from coordination of actions vis-a-vis representations of "objects", or transformations of the "states" associated with series of discrete physical experiences. The critical difference is that logico-mathematical knowledge demands that a pair (or set) or physical objects not be defined by the temporal-spatial (perceptual) similarities, but rather by invariant relations among or between objects.

Figurative and operational processes represent two types of functional structures necessary to account for knowledge acquisition. Figurations are defined as those action schemata that apprehend, extract and/or reproduce aspects of the prior structured or organized physical and social environment. Such action schemata include components of perception, speech, imagery, and memory. Figurations and associated acts are based on physical, as contrasted to logico-mathematical, experience and constitute the "empirical" world. Empirical truth is no more than the "representation of past experience in memory".

Operations do not derive from abstractions from objects and specific events; rather, operational knowledge is derived by abstractions from coordinated actions relevant to those events. Thus, operations are those action schemata that construct "logical" transformations of "states". Such "logical" systems of transformations operate either upon representations of events, or on the cognitive system's own logical operations, i.e., reflexive operations.

Figurative and operative structures are two parallel streams with their genetic or developmental origins in the same source (Piaget, 1967, 1968, 1970a, 1970b; Piaget and Inhelder, 1971b)-- the sensori-motor structures. Logical (operational) structures are not immediately generated by the figurative schemata alone; i.e., not from perception, memory, etc. Reciprocally, figurative structures do not derive from operative schemas but from the representations of past states of events derived from physical experience. And most importantly, figurative structures do not derive from each other, but have unique bases in sensori-motor schema. Imagery, for example, is a derivative of deferred sensori-motor imitation (Piaget, 1951, 1952; Piaget and Inhelder, 1971b) and not perception.

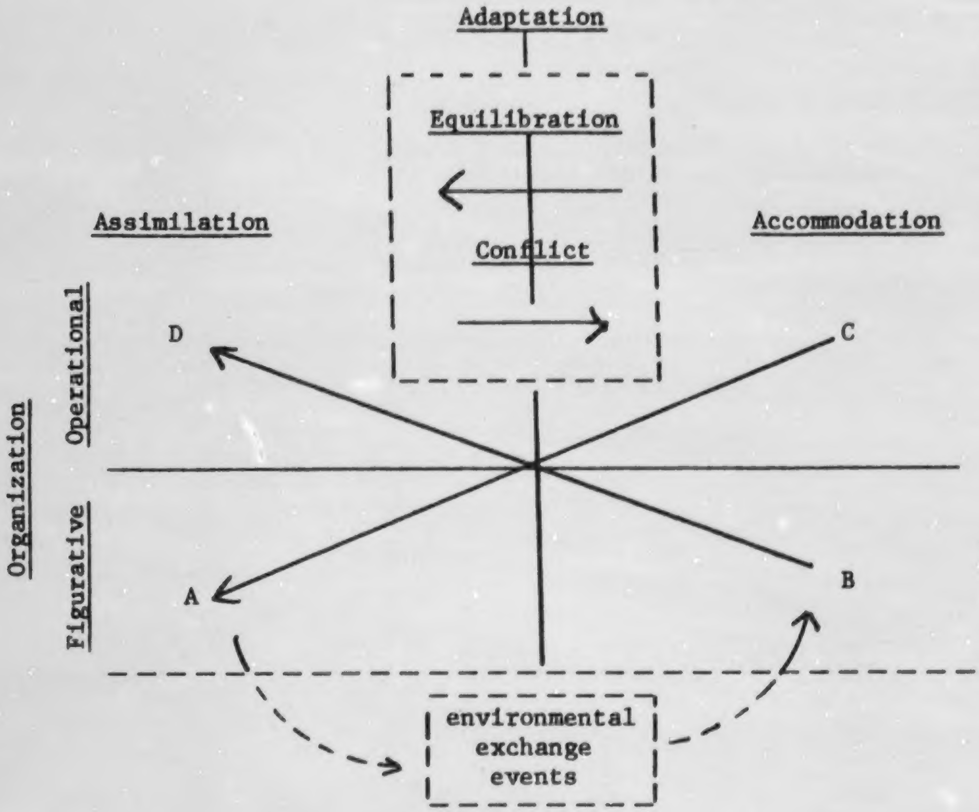
The postulation of these quite different levels of functional structures is one of the cornerstones of Piaget's theory of knowledge acquisition and cognitive development (cf: Furth, 1969). The source and function of each structure is theoretically distinct. Operative structures derive from abstraction from coordinated actions. Figurative structures derive from sensori-motor and perceptual activity. Operative structures produce "logical" transformations (conservation of invariants) whereas figurative structures reproduce sensory-perceptual consequences of externalized or "environmental" configurations. The variant operative structures of the intuitive, the concrete, and the formal levels form the discontinuous sequence of stages of cognitive development. Figurative structures are static and depend directly upon the data of experience (sensory-perceptual consequences of stimulation). Piaget makes the fundamental assumption that all knowledge acquisition activity is constructive, but the construction of figurative representations is quite a distinct process from the constructive activity at the operative level.

Logically, there are three possible relations between the figurative and operational structures (Langer, 1969). First, they may be unrelated. If so, as mentally segregated functional structures, they do not set limits on the functioning and development of each other. Second, psychological phenomena might be reduced to one of the types of structures. Langer (1969) suggests that subjective idealists try to reduce psychological phenomena to assimilatory operations. There are many theorists who try to reduce all mental phenomena to accommodatory figurations and the naive realists propose all knowledge if figurative (e.g., perception is knowledge; Michotte, 1963; Garner, 1962). Third, and the one proposed by Piaget is that of partial communication between figurative and operative structures within the constraints of the assimilation and accommodation processes.

The relations and the potential form of interactions of the components of adaption and organization discussed above are schematically presented in Figure 1 below.

Figure 1

Relations of Two Invariant Processes of Adaptation and Two Types of Cognitive Structures


Adaptation
Equilibration
Conflict
Assimilation
Accommodation
Organization
Operational
Figurative
D
C
A
B
environmental exchange events

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Langer (1969) has examined the organizational and developmental (i.e., transitional) impact of accommodatory figurations on assimilatory operations (B, Figure 1). This is equivalent to asking how does the child mentally extract and/or represent empirical information about physical and social objects and the consequences of that empirical activity for the construction of logical concepts. Imitation of an observed event, comparison of one's predictions with the perceived outcome of a physical manipulation, comparison of an observation or appearance (i.e., immediate experience) with the way things have been constructed and externalized, represent different modes of introducing internal conflict and cognitive-structural change. Generally, his findings are confirmatory, but not definitive with respect to the Piagetian hypotheses. In any case, if the development of each type of functional structure has implications for, but not direct causal effects upon, the structure and development of the other, current paradigms for the study of learning mathematical concepts will require considerable modification. The work of the Geneva group mentioned earlier, concerning for example, memory (A, Figure 1) and Langer's (1969) analysis of the impact of accommodatory figurations (i.e., imitation,etc.) on assimilatory operations represents beginnings in this direction.

Analysis of learning, in the context of Piagetian theory, poses requirements for more detailed empirical analysis than has been generally recognized. On the one hand, researchers attempting to assimilate Piaget to their own conceptual structures concentrate on experimental procedures whereby the subject is required only to remember event contingencies (e.g., response-reward associations or "a" follows "b", follows "c" or similar figurative structures)(Bruner, et al. 1966). Such procedures certainly produce change in "behavior" (e.g., Gelman, 1969; Mehler and Bever, 1967; Bever and Mehler, 1968); however.

failure on transfer tasks, and a lack of persistence of task solution over time indicates that a figurative process underlies the change in performance. On the other hand, the accommodators (i.e., those more favorable toward Piaget's theory) often fail to generate experimental paradigms that adequately differentiate between the figurative and operational knowledge (Wallach and Sprott, 1964) or assume that "external disparity" (appearance vs. "reality") is sufficient to establish disequilibrium or conflict between logical necessity derived from the operational structures and perceptual pregnance (cf: Bruner, 1966). Situations designed to establish disparity between the child's predictive judgment of the outcome of a transformation and his observation of the actual outcome may, in fact, generate little or no cognitive conflict. A most parsimonious explanation of many "negative" findings in training studies is that such disparity belongs to the experimenter's "reality" and is external to the child's own logical operational system.

Implications for Learning and Instruction

In some form or other, the goals of American educators have always been stated in terms of "optimizing" the intellectual, social, or alternative aspects of development of individual children. Whatever imperatives such goals imply, the educational and instructional processes must be based upon an understanding of the nature of psychological development of children. Whether we want to produce individuals who will strive to maintain the status quo, individuals who desire and accept change, people content to be technologists, or problem solvers; it is necessary to understand the basic processes of child development and the conditions that permit "quality control of the product".

The issue is important because science can only yield "what is" and not what "ought to be." We are fortunate, in one sense, that the sciences of psychology and of pedagogy are young and imperfect. The proposed models and methods for educating young children are no less imperfect and are influenced as strongly by current social thought and individual philosophical biases as by an understanding of the laws of psychological development. Such a state of affairs, while producing wasted efforts, spurious claims, and more rediscoveries than discoveries, may hopefully provide time for the development of articulated sets of societal goals for education.

The best that can be hoped for, under the current conditions of our knowledge, is development of preliminary "models" for instruction. Such models can provide, at least, a schematic set of principles and guidelines for constructing a learning environment consistent with the admittedly inadequate theories and knowledge of psychological growth. However, we should try not to violate recent advances in theory and known laws of child development.

Piaget, until recently (Piaget, Jean. Sciences of Education and the Psychology of the Child, 1971), declined to generalize his theory to specifics for educational practice. His theory of knowledge acquisition has contributed to clarification and integration of a set of propositions about phychological development, many of which have a long history in child psychology and education. If we accept his theory of cognitive development, several deductions concerning the construction of "optimum" environments can be generated. A modest attempt in this direction has been made at the University of Georgia for the Follow Through Program (Smock, 1969). Though the basic propositions of the model are not inconsistent with Piaget's thinking about knowledge acquisition, the interpretation is that of the modeler. It is influenced, therefore, by numerous sources of bias, misunderstanding, and distortions that are inevitable under conditions where abstract theoretical concepts are not represented in unequivocal abstract or logico-mathematical terms.

We start with the general proposition that the child is not a passive recipient of stimulation, nor can external reinforcement be considered a primary factor in learning and behavioral change. Further, the introduction of "mediation responses" (verbal or otherwise) is not able to account for the complexities of observed changes in behavioral organization during the course of psychological growth during childhood. Many psychological theorists have adopted, in one form or another, the idea that human organisms actively respond to their environment and that the patterning of these responses reflects a "plan" or "set of cognitive operations". In other words, the child "interprets" environment input, and the interpretations are controlled by his capabilities for generating rule systems for coordinating and transforming the input to "match" a scheme, plan, or a mental operational structure. Analysis of the "rule systems" characterizing cognitive development, thinking, and learning, requires specifications of the properties of, and antecedent conditions for, selection and structuring of the consequences of environmental events (mental representation/figurative knowledge) and of the mental actions (operative knowledge) necessary for coordination and transformation of those representations. The study of the development of rule systems defined as such is coincident with the systematic investigation of the "inherent logic" of development of operative and figurative thought processes.

Intelligence, first of all, is considered no more, and no less, than biological adaptation. Adaptation at any level of complexity reflects "intelligent" activity. "Knowledge" consists of two types of functional structures (figurative and operative) that give rise to invariants in organism-environment relations. These invariants are derived from abstractions from objects (physical experience) in the first case, and from coordinated actions (logico-mathematical experiences) in the second. Intelligence, then, refers to both types of cognitive learning and is defined in terms of functions (i.e., thinking or reasoning) rather than content (i.e., words, verbal responses, associations, et cetera). Analysis of conditions for cognitive learning and development must begin with the identification of components of behavioral organization (structure) that reflect particular coordinated action-modes of the child as he is confronted with changing intrinsic (maturation and prior cognitive acquisitions) and extrinsic (physical and socio-linguistic) factors.

Cognitive structures or systems of coordinated (mental) actions proceed through invariant stages of structural change with ontogenetic development. The successive differentation and hierarchical integration of these cognitive structures permit the individual to cope with increasingly comples social and physical "realities". The process of cognitive development involves the changing characteristics of transformational rule systems (virtual and/or cognitive operations) characterizing the child's mode of adaptation. Neither the maturational structure of the organism nor the "teaching" structure of the environment is the sole source of reorganization; rather, it is the structure of the interaction (exchange events) between the child and the perceived environment that provide the basis for intellectual development.

Optimal conditions for structural organization and reorganization require: a) an optimal degree of discrepancy between perceived environmental/demand structures (i.e., perceptual activity, images, memories) and cognitive operational structures; and b) social-learning conditions that demand "spontaneous" or "constructive" activity by the child.

Several implications for the construction of theoretically appropriate learning environments are implied in these general principles. First, structural change depends upon experience but not in a way that traditional learning theorists conceive of experience or learning interpreted as pairing of specific objects and responses, direct instructions, modeling, etc. Rather, the functional genetic view holds that the cognitive capacities determine the effectiveness of training. For example, ability to solve class inclusion problems implies that the child already has the requisite single and multiple classification operational system for classes (i.e., combination, reversibility, etc.) in addition to appropriate information selection, storage, and retrieval abilities. At the same time, while experience is necessary for developmental progress, and appropriate enrichment of the environment can accelerate such development, experience cannot change the sequence, structuring, or emergence of action modes in the process of developmental change. In other words, organization of experience is not provided solely by the environment nor solely by the structures intrinsic to the child.

Second, the structure of learning environment must be considered relative to two frames of reference; that is, in terms of the operational systems controlling the child's interpretation of "environmental" events and the content to be learned.

Operational systems are expressed behaviorally in the coordinated actions of the child confronted with changes in his physical and social world. For example, the mental operations of associativity, reversibility, et cetera are inferred from the manner in which the child attempts to solve problems involving regular environmental contingencies or causality understanding of spatial relations, arithmetic and the other substantive areas, such as science or mathematics. Each must be analyzed in terms of their own logical sequence and commonalities with other content areas. Certain concepts in the physical science, language, and mathematics, for example, may have an inherent sequence and structure. Thus, certain concepts may be necessary precursors to subsequent understanding of higher order concepts. Optimal educational conditions require, then, thorough understanding of the psychological-cognitive capacities of the child as well as the sequential structuring of concepts within a particular curriculum area.

Third, the striving for equilibration between assimilatory and accommodatory processes under both intrinsic and/or extrinsic pressures underlies the adaptive process. Optimal conditions for structural reorganization, learning in the broad sense require disequilibration. This condition is met when there is an appropriate "mismatch" between the cognitive capacities of the child and the conceptual demand level of the learning task. Too little or too much "pressure" may result in over-assimilation or over-accommodation respectively, and not promote cognitive-developmental change.

Fourth, facilitation of learning requires analysis of two levels of cognitive functioning -- figurative and operative processes. The first is most emphasized by those theorists, particularly behaviorists, recommending a direct tuition approach to instruction. The operational theory of intellectual development does not deny the value of "provoked" learning (i.e., through imitation, algorithms). Rather, such learnings are considered limited because of lack of generalization or transfer to new situations and because the basic intellectual processes concerned with problem solving and reasoning are not significantly affected.

While there is some doubt that much acceleration of structural reorganization is possible through environmental enrichment, early childhood education should provide opportunities for utilization of relevant cognitive operational structures. Generalization of conceptual learning across content areas rather than the building of specific knowledge and skills (e.g., a large vocabulary) should be emphasized since the latter cannot directly accelerate operational system change and may, in fact, retard development of these "deeper" competence structures.

In any case, the nature and variety of the child's "exchanges" with the environment need to be considered in educational planning. The nature of the interaction refers to the relative emphasis on ontogenesis (self-directed) as contrasted to exogenesis (environmental or teacher-directed) structure of the learning environment. The functional genetic position can best be summarized in the old adage -- "you can lead a horse to water, but you can't make him drink - unless you feed him salt". Thus, the task of the teacher is to engineer an educa tional environment consisting of curriculum materials, social interactions, and directed activities that provide appropriate "salt" for each child. Sequentially structured curricula should be designed to provide an optimum degree of structure and conceptual level to permit an appropriate balance of assimilatory and accommodatory activity.

The variety of interaction or enrichment refers to the types of structured curriculum contents relevant to the child's physical, social, and symbolic experiences. The interlocking nature of substantive curriculum areas makes it possible to provide a variety of experiences relevant to acquisition of the cognitive "products" that provide representation of the environment, such as memories, vocabulary, or symbols and, at the same time, to facilitate the development of coordinated rule systems associated with cognitive operational development. For example, analysis of the visual environment (attention or observational skills) as well as cognitive operational structures (e.g., conservation of area) can be emphasized in science, social studies, mathematics and art.

The engineering of an educational or "learning environment" based on the preceding consideration necessarily involves the development of specifications of: 1) the child's cognitive developmental level; 2) the physical structures, including curriculum materials; and 3) the social or interpersonal structures. The organization of these "elements" should be such that equilibration, between different cognitive systems and/or between intrinsic functional structures and "environmental" structures, is achieved. Thus, sequentially structured sets of curriculum materials and of social interaction situations are necessary to provide the "pressure" necessary for learning and adaptation. A variety of specific learning environments needs to be available to maximize the probability of each child's finding activities that attract or "trap" him into interacting with the physical and social environment at both the behavioral and symbolic levels of language and mathematics in creative and spontaneous ways be it through art, role playing or music. Finally, the physical and social environments should be arranged so that considerable freedom of movement, within the structure of a variety of contents, is possible; i.e., "a modified open-structure classroom". A careful balance between relatively high and low structured learning situations and between group and individual learning activities should be maintained.

The Mathemagenic Activities Program, a model developed in the context of enriching the educational environments of economically deprived children,[3] is based on three explicit principles derived from the considerations discussed above. Specifically, the MAP principles of change -- whether the target for change is the individual organism (child) or a complex social system (e.g., Local Education Authority) are based on the above assumptions concerning the role of experience in learning and development. First, the source of motivation to change is provided by a discrepancy (disequilibration) between different conceptual systems (ideas) and/or between previously acquired conceptual systems and environmental task demands. Thus, an *appropriate mismatch* (M) is necessary to generate exploratory activities and insure the individual has the prerequisite conceptual basis for learning higher order concepts.

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[3] Mathemagenic Activities Program: A Model for Early Childhood Education, prepared by C. D. Smock (A Preliminary statement of the conceptual basis for the Mathemagenic Activities Program for the Follow Through Program appeared in Terminal Report, Research and Development Center for Early Educational Stimulation, University of Georgia, August,1970).

Second, since coordinated actions (practical and mental) are the bases for knowledge acquisition, the learning environment must be structured so that specific task demands include appropriate practical, perceptual, and mental *activity* (A).

Third, the learning environment must include provisions for personal *self-regulatory* (P) constructions. Knowledge acquisition involves construction of invariants from properties of objects (physical experience) and from the child's actions on objects (logico-mathematical experiences). Optimal conditions for facilitating new "constructions" (concept learning) involve a balance between tasks that are highly structured (in which the child merely "copies" or imitates the correct solution) and tasks that permit the child to generalize and discover new applications of his concepts. Practically, self-regulation implies a *variety* of task *options* available to the child; the number of options may well vary with the nature of the task and many other factors. MAP proposes, however, that options -- in terms of level of task difficulty, mode of learning, and choice of activity -- are necessary ingredients of developmental change, whether the target be a child, a teacher, or an educational system.

The implied educational model requires significant changes in the teachers' role definition and teaching strategies and tactics. The need for sensitivity to the child's capabilities, and the structuring of learning situations that promote self-regulated, "constructive" knowledge acquisition, together with thorough acquaintance with available technological aids, require an "educational engineer" in the best sense of that term.

MAP Publications--September 1975 complete Update

Research Reports

1) Discovering Psychological Principles for Mathematics Instruction
Charles D. Smock

2) The Use of Mathematical Models for Describing Sequential Search Strategies in Elementary School Children
Robert Lissitz, Philip Lewis, Patrick Brady

3) Measures of Language Comprehension: Relations to Performance
Rodney Cocking

4) Models for Performance in Syllogistic Reasoning
Paul Silverman

5) Elementary and Middle School Children's Comprehension of Euclidean Transformations
Richard Kidder

6) Ethology's Modest Contribution to the Assessment of Education
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7) When Do We Know What a Child Knows?
Irving Sigel

8) What Should We Measure When We Measure Linguistic Competence?
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11) A Survey of Cognitive Abilities of Children in Different School Programs
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14) Epistemology and Education (A Collection of Essays)
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15) Development of Social Relations: Role-Taking Skills and Classification Abilities
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Research Reports

16) Radical Constructivism & Piaget's Concept of Knowledge
E. von Glasersfeld, June 1975

17) The Concepts of Appearance, Disappearance, and Reappearance: A Developmental Semantic Analysis
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18) On a Model for Learning and Teaching Mathematics
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19) A Constructivist Model for Instruction
Charles D. Smock, June 1976

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The Constructivist Epistemology in John Dewey, Jean Piaget, and Cognitive Developmental Psychology

by

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Abstract

Theories are judged by the consequences of their application; however, when theories are misinterpreted, the consequences can only support, or fail to support, the misinterpretation. Jean Piaget's theory of cognitive development makes explicit an epistemology which has been called constructivist; John Dewey has been described as holding a similar theory of knowledge. In this paper I argue that the peculiar status of the term environment in the theories of Dewey and Piaget has not been generally understood; further, the emphasis on the similarities between the theories has resulted in the neglect of some differences between the two theories.

Dewey's theory of knowledge is presented as an account of the dynamic conceptual organization of purposive interactions with experienced sense data. This account of an active knowing process is presented within the context of the philosophical implications of Heisenberg's Principle of Uncertainty. The result is a distinction between the world as it is, which we cannot know, and our knowledge of the world which we construct. Piaget's theory differs from Dewey's in terms of its "structuralism," and the distinction between figurative and operative processes. The similarities between the two theories are discussed in terms of the dialectical process of experiencing the consequences of operations. A number of misinterpretations of the constructivist epistemology are presented at the end of the paper.

The Constructivist Epistemology in John Dewey, Jean Piaget, and Cognitive Developmental Psychology[1]

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Introduction

There are affinities in the descriptions of the acquisition and nature of knowledge which have been presented by Dewey and Piaget. Kohlberg (1969, 1971) and Kohlberg and Mayer (1972), who are developmental psychologists, have noted these similarities. However, their statements of this association neither explore the extent of the similarities or the nature of the differences between the theories of Dewey and Piaget; nor do they make clear the radical nature of the constructivist epistemology on which they rest. Since considerable attention is being given to the cognitive developmental model at present, it is important to try to clarify these issues. This will be attempted by means of an analysis of the constructivist position in Dewey's epistemology emphasizing the role of action and operations in the process of knowing. The similarities between the theories of Dewey and Piaget can be described in terms of their pragmatist analyses of the role of activity in knowing, and also in terms of their accounts of the process of knowing. The differences between the two theories will then be discussed since they have been underemphasized in cognitive developmental accounts of the similarities between these theories. At the end of the paper, a number of interpretive problems will be discussed.

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John Dewey and the Constructivist Epistemology

Dewey (1929) argued that an analysis of the relation between ideas and action at the time of the scientific revolution shows that a radical change had occurred in the implicit understanding of the origins and nature of knowledge. This change pointed to the important role of action in the process of knowledge acquisition. The history of epistemology is a long one, and clearly beyond the scope of this paper. However, it will be helpful to describe the accounts of the origins of knowledge which were prior to the synthesis presented in Dewey's analysis. Did knowledge come from sensation, or from thought, or from some combination of the two? Thus, one view held that knowledge derived from sense data which was recorded in a passive mind. Reality was outside, and we knew reality because our senses received data from it. An alternative view maintained that the world was a chaotic reflection of the pure forms of thought, which alone guaranteed truth. In this case, ideas were the structure of the reality which is outside. One synthesis of these two positions was available in Kant's philosophy which postulated that knowledge was possible because of the 'a priori' conceptual organization of sensations. The constructivist account of knowledge and knowing differs from each of these positions in its emphasis on the role of the activity of the knower in his organization of and integration of his sensations and ideas. A second difference is the denial that the validity of knowledge can be verified by checking with 'reality'; this is entailed because 'reality' is itself a construction.

The following quote can serve as an introductory statement to the constructivist epistemology:

> "The scientific revolution came about when the material of direct and uncontrolled sense experience was taken as problematic; as supplying material to be transformed by reflexive operations in known objects." Dewey (1929, p. 258).

The implication of this is that what is known is the result of the organization of interactions between the individual's ideas and his experience of his surroundings. Further, while Dewey was clearly referring to sense experience resulting from interactions with 'material objects' in the quote just given, it is clear that this model of knowing was not to be restricted to such objects, but rather was intended as a general account of the process of knowledge acquisition. There is another reason that this quote is useful in introducing the constructivist epistemology; this is the identification of the process of knowing with the process of science. In fact, Dewey (1929) described his epistemology as a theory of experimental knowing.

Dewey's account of the relation between activities and ideas cuts across the separate status given these terms in both classical epistemology and ordinary language usage. As Dewey pointed out, the common usage of the words 'ideas' and 'activities' grew out of an epistemology which developed prior to the scientific revolution. Thus, the way in which we separate ideas and action in our everyday language affects our understanding of their interrelatedness. One way of understanding the problem is through an analysis of the early development of the child's thought. Ceccato (1961), who is a cyberneticist, has argued that our understanding of what it is to know was distorted because the activities involved in observation are acquired early in infancy, so it is difficult for adults to observe these activities. Studies on the development of cognition and perception were needed to clarify the dynamic relation between action and knowledge. Another argument presented by Ceccato was that philosophy first began to ask questions about the nature of observation during a period when the tradition of speculative philosophy was firmly established. Under these conditions observation was studied as the observation of objects, which were naturally assumed to exist prior to their observation. Thus, it was not possible to question the independent existence of the reality which is known.

Dewey's analysis of the relation between ideas and action grew out of Peirce's writings on the clarity of ideas. "It appears, then, that the rule for attaining the third grade of clearness of apprehension is as follows: Consider what effects, which might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object" Peirce (1881). Also, while considering the continual growth of knowledge and the emergence of new concepts, Dewey described ideas as tentative hypotheses. These hypotheses were anticipatory plans whose value was discovered in the consequences of testing these plans.

There are two types of operations involved in the actual testing out of ideas, physical operations and symbolic operations. Physical operations are required in order to ascertain the nature of the data received. Thus, activities such as looking, touching, and listening result in sensory consequences. Conceptual searchings through past experience yield ideas which can be used to assess the sensory data. This previous knowledge will also suggest new ideas and new experiments until the additional evidence supports one idea and the problem is solved (Dewey, 1929). The dynamic manner in which data and ideas interact is a central feature of the constructivist epistemology and one which distinguishes it from Kant's epistemology. The following quotes are useful in understanding Dewey's position:

> "Original objects of experience are produced by the natural interactions of organism and environment, ... The distinction between sense data and interpretive ideas is deliberately instituted by the process of inquiry, for the sake of carrying it

> forward to an adequately tested conclusion, ... Hence the material selected to serve as data and as regulative principles constantly check one another; any advance in one brings about a corresponding improvement in the other. The two are constantly working together to effect a rearrangement of a new object having the properties which make it understood or known." (Dewey, 1929, p. 173)

Dewey's use of the term operations in the quote given above refers to Bridgman's operationism. Bridgman was a physicist who was writing about the epistemological consequences of post-Heisenberg physics, and he made explicit the pragmatists' account of the relation between activity and concepts. Operations were defined in the following way:

> "To find the length of an object, we have to perform certain physical operations. The concept of length is therefore fixed when the operations by which length is measured are fixed; that is, the concept of length involves as much as and nothing more than the set of operations by which length is determined. In general, we mean by any concept nothing more than a set of operations; the *concept is synonymous with the corresponding set of operations*. (Bridgman, 1927, p. 5) (Italics in original)

It is necessary to emphasize that Bridgman's intention was to try to clarify the nature of the relation between activities, words, and experiences. This aspect of Bridgman's operationism was emphasized by Dewey, but has been underemphasized by psychologists who have found the operational definition a useful tool ensuring methodological objectivity. It is important to clarify this latter point because otherwise the use of the word operation will consistently suggest the wrong meaning. For an example of this standard transformation of Bridgman, consider Stotland and Canon (1972):

> "The thinking behind such an approach maintains that we mean by such a concept nothing more than a set of operations by which the concept is measured or manipulated in some fashion (Bridgman, 1928). Thus, the concept of intelligence would be defined as whatever it is that standard tests of I.Q. test or measure." (p. 2)

In this quote Stotland and Canon have used the operational definition to standardize method. This usage of the term operation is useful, though it is a usage which is not cognizant of the other technical usage of this term as it is found in Dewey's writings. Generally, psychologists have failed to understand the manner in which operations relate to concepts. In strong contrast, Dewey defined concepts as definitions of the consequences of operations.

Bridgman's concern with specifying operations grew out of his dissatisfaction with the manner in which words describe experiences. He felt that language descriptions make experiences appear static, in a manner which is not true to the experience. It is important to remember that our experience is not composed of static items and that it is for convenience that we talk as though this were so. This convenience results from the correspondence between aspects of present experience and aspects of past experience. However, this convenience may present problems since one is dealing with a construction which doesn't reproduce experience but only certain features of it. The only recourse which scientific epistemologists have is to test statements which purport to be knowledge statements by translating them into testable operations, because meanings are determined by operations.

When concepts are defined in terms of operations, their adequacy can be assessed with reference to the consequences of these operations. When there is a problem in the data one tries to account for it, and whether it is a problem in psychology or in driving one's car, the process of thought is the same. For example, in reflection one forms an idea about the problem and its solution, and then translates this idea into testable operations. The consequences of these operations either confirm or fail to confirm the idea on which they were based. In the case of well known operations, such as driving a car (for many people), the reflection will be quite automatic.

The notion of testing ideas in terms of their consequences, as opposed to testing them with reference to an antecedent reality, was not possible without an understanding of the relation between purposive activity and knowing. Dewey (1929) maintained that this understanding was absent from epistemologies prior to that of Peirce (1881), whether they were realist or idealist, and irrespective of whether they assumed man to be a passive recipient of environmental information or an active synthesizer.

The ideas which we know are no more than means by which we organize our experience of our environments. It is often difficult to remember that it is logically incorrect to assume that these ideas reflect an objective reality. For this reason Dewey warns against 'reifying' ideas, and exhorts thinkers to try to remember that sense data are taken from our experience of the environment, noting how different the history of epistemology would have been if this had been noticed earlier.[2] It is the directed or purposive nature of our knowing process which limits our descriptions of phenomena; since the ideas are constructed within the constraints of a particular problem, or particular problems, it is always possible that a change in the nature of the problem will entail a corresponding change in the ideas used to organize our experience of the phenomena. Another limitation to our ideas stems from the changing nature of our experience of the environment. This follows from what has been said about the nature of uncontrolled experience, and about the importance of controlling our experience in order to organize it into something known. So our ideas may suddenly be found inadequate because of some aspect of our experience of which we are initially unaware. When our ideas are found to be inadequate, we must construct new ones by formulating hypotheses about the inadequacies and testing them operationally.

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[2] It should be noted that Berkeley, the Irish 'British Empiricist,' recognized this (e.g., Of the Principles of Human Knowledge, section 38).

In the constructivist epistemology our knowledge of reality is recognized as being our organization of our experience of reality. In this context the problem of the validity of our concepts, which traditionally was one of discovering the relation between knowledge and reality, is replaced by the problem of discovering how to test our ideas and intuitions. Thus, within the constructivist epistemology ontological questions are unanswerable and all knowledge is considered tentative and not final.

The view that knowledge is not to be assessed by a comparison with an antecedent reality, but by the consequences of planned operations is the central theme of The Quest for Certainty. The irrelevance of ontological questions is apparent in Dewey's (1929) discussion of Heisenberg's Principle of Uncertainty. From this Principle it followed that either we can decide to control the velocity of particles and incur inaccuracies with respect to mass, or we can control the mass of particles and incur inaccuracies with respect to velocity. In each case we have to recognize that our concepts slightly distort the phenomena described. This is a very serious problem if we have assumed that our concepts reflect reality in an objective sense. And it is the common sense view, and I believe the view of most psychologists, that our concepts do reflect reality in this objective sense.

Dewey described the epistemological consequences of Heisenberg's Principle in the following manner, consistent with the modern view of Capek (1961):

> "Since either position or velocity may be fixed at choice, leaving the element of indeterminacy on the other side, both of them are shown to be conceptual in nature. That is, they belong to our intellectual apparatus for dealing with antecedent existence,

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> not to fixed properties of that existence. An isolation of a particle for measurement is essentially a device for regulation of subsequent perceptual experience." (1929, pp. 202-203)

The notion of regulating subsequent perceptual experience for experimental purposes is similar to the theme of Powers' recent book Behavior the Control of Perception (1973). Unfortunately, space does not permit any detailed analysis of parallels between the constructivist epistemology and Powers' cyberneticists approach in this paper. For a description of these relations, the reader is referred to von Glasersfeld (1974) and to Silverman (1974).

Piaget and Dewey, similarities and differences

Similarities between the theories of Dewey and Piaget have been listed by Kohlberg (1969), and described as assumptions of the cognitive developmental model. It should be made clear that these assumptions have been derived from experiment and confirmed in experiment; they are not assumptions in the sense that they have been arbitrarily chosen. Before listing some of the similarities, I would like to call attention to the loose manner in which the word 'environment' is used by Kohlberg (1969), a sense in which it is clearly possible to speak about the interaction between the organism's structure and the structure of the environment. However, the philosophical laxity of some cognitive developmental psychologists is a matter for the next section. Kohlberg's list of similarities has been adapted to 'fit' the position of this paper and included: 1) development involves transformations of structures which must be explained holistically; 2) development is the result of the processes of interaction between the organism's structure and the structure of the experienced environment; 3) cognitive structures (schemata and schemes) are always action structures irrespective of whether the action is a physical or a cognitive operation; 4) the direction of development is always toward greater equilibrium in the interaction between the organism and its experience of its environment; 5) the apparent emphasis on cognition is resolved by maintaining that the affective domain is parallel to the cognitive.

The historical basis for Kohlberg's list of similarities is clearly stated by Piaget (1970b), a portion of this book which was originally written in 1935.

> "On the one hand the work of the pragmatists had revealed the role of action in the constitution of all mental operations, and of thought in particular; on the other, the science of mental development, or genetic psychology, had increased considerably in scope, particularly with the work of Stanley Hall and J. M. Baldwin. These two trends found their exact point of intersection in John Dewey,...." Piaget (1970b).

Some of Kohlberg's similarities will be familiar from the preceding account of Dewey's epistemology, and they are not independent as they all refer to the origins and nature of knowledge. Both Dewey and Piaget describe 'knowledge of reality' as a personally constructed organization of the sense data attended to; an organization which is tested each time it is used and which becomes more adequate as a result of modification after such use. We have seen that there is a denial that this knowledge can be correctly described as a reflection of the reality outside the organism in Dewey's case, and that this position came about as the result of his reflections on the implications of operationism, Peirce's pragmatism, and the Principle of Uncertainty. In Piaget's case there is a corresponding denial that knowledge is a reflection of reality; and an assertion that knowledge results from the organism's activities. The limited nature of the concepts which we construct to relate to our environment, in Piaget's theory, is held to be vindicated by Gödel's theorem. This theorem states that formal systems, which are at least as complex as arithmetic, are necessarily incomplete. Completeness is proven when it can be shown within the system that no logically incompatible statements can coexist within the system. What Gödel's theorem implied, and Gentzen later proved, was that in order to demonstrate completeness in any formal system one had to use principles of reasoning from a more complex formal system (Piaget, 1970c, p. 32). Piaget's inference is that knowledge itself can no longer be considered complete, since each new refinement will require justification at a newer and higher level. Smock (1973) has noted that this incompleteness stems from Piaget's recognition of the implications which the Theory of Relativity holds for psychological theory.

However, the recognition of this type of limitation does not impede the continuing construction of more adequate concepts, with which to relate to our environments, through the systematic application of the scientific method. This notion is as thoroughly Piagetian as it is characteristic of Dewey. In Piaget's theory conceptual development comes about through the equilibration process, which is adaptive, self-regulated, and dialectical. In other terms what is implied is that one's understanding of an experienced situation is limited by one's current understanding; new events are assimilated by existing structures. Aspects of these environmental situations which are different from current understandings of them, but within a match-mismatch range (Hunt, 1961) will be recorded by means of accommodations to the existing structures. The parallel with Dewey's account of experiencing is striking, especially the dialectical nature of experiencing in both theories.

Dewey (e.g., 1934) described experiences in terms of interactions between any live creature and its experience of its environment. These interactions consist of doing something and in consequence undergoing something. What is done and undergone will vary depending on the content of the experience, whether it is lifting a stone and assessing its suitability for some purpose, or selecting an appropriate statistical technique and checking that the assumptions aren't being violated, given the nature of the data. In this context the function of intelligence is defined by Dewey to be that of perceiving the relation between what is done and what is undergone as a result of a particular action. This action process of doing and undergoing continues until there is a mutual adaptation of the conceived ideas and the perceived object, and the experience comes to a close. Thus, when something is finished in a satisfying way we are prone to say that we have had 'an experience' because what has been done and undergone has been completed and the result has coherence.

A consideration of the levels of possible operations in the two theories shows that Piaget's sensorimotor, concrete, and formal operations have parallels in Dewey's theorizing. Dewey's (1929) discussion of formal logic as a system of operations which act on other symbolic operations (p. 154) is clearly identical to one way of describing Piaget's distinction between formal and concrete operations; and Dewey's discussion of eye-hand coordinations (1933, p. 206) is a good outline of the carefully detailed developmental-observational studies of this phenomenon which were performed by Piaget. Other parallels which could be mentioned are Dewey's separation of thought and language, stressing the primacy of operations (1933, p. 183); and Dewey's analysis of play as activity pursued for its own sake (1933, p. 212). (Dewey [1933] was first published in 1910.)

There are two aspects of Piaget's theory which are different from Dewey's theory; these are the distinctions between figurative and operative processes, and what has come to be known as Piaget's structuralism. In reference to structuralism, Dewey made no attempt to try to discover an underlying logic of the process of coordination of the cognitive operations. In Piaget's theory, figurative and operative structures have different origins and coordinate experienced sense data at different levels. Figurative structures originate in sensorimotor schemata; and operative structures, which originate in the schemes of the concrete operational period, coordinate the figurative structures. Piaget (1969, 1970a) has discussed the difference between the two types of structures in terms of the intentional transformational quality of operative coordinations. Thus, figurative cognitive functioning, which includes perception, imitation, and imagery, does not transform or seek to transform sense experience actively. The function of figurative operations being to represent 'reality' by means of past organization of sense experience.

It is very difficult to eliminate the possibility of misinterpretation; perhaps it is impossible. The ambiguity with which Piaget obscures his epistemology has been pointed out by von Glasersfeld (1974) and is very clearly ambiguous in the discussion of figurative cognitive functioning (Piaget, 1970a, p. 717). When Piaget addresses representation of reality in figurative operations, it is very easy to understand him to mean that the structure of the environment matches the structure of cognition. This position would imply an empiricist epistemology, and as such would be quite inconsistent with other parts of his theory. From the point of view of this paper the constructivist epistemology, in both Piaget's theory and Dewey's philosophy, implies that what one knows about reality 'out there' is a personal construction. It is a personal construction which we can communicate effectively with other members of our species. However, it is a personal organization which cannot be described as an objective reflection of reality.

Interpretive issues

The potential for misinterpreting, or not being aware of, the implications of the constructivist epistemology is obvious; and such misinterpretations are, I believe, quite general. One example mentioned earlier is Kohlberg (1969) who described development as the result of processes of interaction between the organism's structure and the structure of the environment. Similar laxity is found in a recent cognitive developmental account of the literature on imitation theory and research, Kuhn (1973). Kuhn referred to and adopted Kohlberg's (1969) idea of the environment. Again, Riegel (1973a) argued for an extension of Piaget's theory to include the historico-cultural context of development. Riegel's position was that Piaget's theory, at least in the 50's, overemphasizes the individual's role in cognitive development and underemphasizes the social context of development. For example, Riegel stated: "Development procedes through dialectical interactions between psychic activities and their biological and outer sociological interactions" Riegel (1973a). In each of these cases, the environment is interpreted as, or referred to as having an organization independent of the subject. However, psychologists are in good company when being loose philosophically because Piaget himself is treacherously ambiguous in his usage of the term "environment." For another example of this ambiguity, consider the following recent statement by Piaget (1972) on the relation between the subject and the object:

> In short, these pages contain an account of an epistemology that is naturalist without being positivist; that draws attention to the activity of the subject without being idealist; that equally

> *bases itself on the object, which it considers as a limit (therefore existing independently of us, but never completely reached)*; and that above all sees knowledge as a continuous construction:... (Piaget, 1972, p. 17). (my italics)

This unqualified reference to the "object" is confusing; however, I think that it is important to note that this quote is consistent with the view that while the "object" is independent of us, our knowledge of the object can only be described as the result of our intentional interactions with our experience with it.

Another important interpretive issue concerns the need for a final period of dialectical operations (Riegel, 1973b). This argument finds in the development of increasingly adequate logical competence a corresponding alienation from dialectical experience. Riegel qualified this interpretation as being true for Piaget's 'middle period,' that is his theory as represented by *The Psychology of Intelligence*. This present paper has emphasized the identity of cognitive process in Dewey's philosophy and Piaget's theory. Therefore, I think that it is most important to point out that the dialectical nature of the theory derives from the dialectical nature of the equilibration process. So development is described in terms of reaching ever more adequate forms of equilibrium between cognitive structure and the individual's experience (which assimilates this experience in terms of existing structure). To argue that the dialectical nature of experience fades away as the structures become more logical is misleading. For example, Piaget (1968) described the equilibrium of the developing structures in middle and late childhood as more logical in the sense that they are more mobile. That is, these structures (of concrete and formal operations) emerge because they are better at resolving the dialectical interpenetration between the ongoing experience of the child and the existing cognitive structures. It is important to note that this discussion (the first article in Six Psychological Studies) was originally published in 1940 and so is representative of Piaget's 'middle period.'

In summary, a recent paper by Wohlwill (1973) has stressed the importance of recognizing, and taking steps to correct, the loose usage of the words 'environment' and 'experience' by psychologists. It is hoped that this present paper will contribute to a more general recognition of the peculiar status of 'the world' within the constructivist epistemology, because it is this odd status which is the major contribution of this theory to epistemology. Within this present context, it is only possible to refer to knowledge of the environment, or experience of the environment, in the context of any particular individual's experience. To neglect this emphasis is to miss a crucial aspect of the constructivist position. That it seems so easy to neglect is perhaps due to current psychology's relative neglect of Dewey, who did emphasize this, and relative preoccupation with Piaget who seems unwilling to emphasize this.

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Believing is Seeing
An Introduction to
Radical Constructivism

An Interview with
Dr. Charles D. Smock

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at the
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THOMAS C. O'BRIEN: I am happy to introduce Professor Charles Smock from the University of Georgia. Professor Smock is a distinguished Piaget oriented researcher who is in the Psychology Department at the University of Georgia. He is head of the Mathemagenics Activities Program at the University of Georgia and he is also head of the Follow-Through Program at the University of Georgia. Not long ago he spent a year in Geneva working with Piagetian researchers as a Senior Visiting Researcher. Charles, welcome to St. Louis.

CHARLES SMOCK: Thank you. I'm certainly glad to be here again, briefly, after fifteen years.

O'BRIEN: I'd like to start with just a very broad question. The question is, "what does psychology and psychological research and Piagetian research have to say to education?"

SMOCK: That's a very, very broad question. I think I would answer by saying that education is a very broad term. And, usually, in America when we talk about education, we talk about what happens in a school building. Now as a psychological scientist I've learned that we need to become much more precise in our use of terms, just as one does in mathematics. We need to know what we mean by the symbols. And as a psychologist, I've been very concerned about the meaning of the term "education". What does it mean? Unfortunately, over the years, I've found no statements of goals or objectives at the philosophical level that would allow me to enunciate the goals and objectives of American education in a precise form. (I have come to a certain opinion that I'll be glad to tell you about later, if it's relevant, about what those goals and objectives are. I don't think they're generally publicized very much.)

But when I think about education, I think about learning, because education -- as we think of it generally -- has to do with children's learning. And it seems to me, one of the basic orientation problems American education has had, is its emphasis on instruction in teacher training rather than an emphasis on how children learn.

I think the starting point for education in school has to be a thorough understanding of the nature of children's thinking and the development of children's thinking. So I think psychology could have a great deal to say about the development of children's thinking and I think Piaget's recent rise to stardom in this country is based on the fact that he is interested in children's thinking. He's not interested in teacher training as a priority but rather he has been asking, "How do children think?" and "How does children's thinking develop?" Once you know that, you know how to train teachers. The purpose of education then becomes one of providing opportunities for children to think and to develop their thinking powers.

O'BRIEN: Do you see the role of the teacher, then, as being a helper in the growth of the child's thinking?

SMOCK: A helper and a guide and a caretaker. In much the same way that a mother gives food to her child, a teacher needs to give intellectual food to children. And both mother and teacher need to know a lot. In many ways I think mothers know more than teachers about what food the child needs, because they see the child in the everyday world of experiences. There's nothing artificial about a basketball game or about tossing pennies in the alley. These are real, whereas school too often is an artifical environment to kids. They're always being prepared for something in the future. Perhaps you've read one of the papers by Charlesworth that emphasizes the importance of studying the behaviors of children in naturalistic situations. He points out that with the primitive bushman and his son--who have no school of hunting-- the child's tasks are sequenced exactly so that he ends up being able to do what his father does and maybe even better, because the child learns every step of the process of becoming a hunter. First, he helps the mother build the fire that heats the material in which he dips his arrow. Then he learns about how hot that water has to be. Then he learns how to make arrowheads and how to attach them. Then he learns how much poison to put on the arrow tip and then he goes on mock hunts with his father. So his life is real. There's no preparing for the future. Every step of that is real to the child and real to the father. There isn't the separation of means and ends that we find so much of in American education.

For example, I recall an article whose title I read and I thought, "That's silly." Someone said, "Why Teach Mathematics?" And I thought, "Now, why in the world would anybody ask that question?" As a matter of fact, it's a darned good question--"Why Teach Mathematics?" What use is mathematics to a five-year-old child? And until you can find that use in terms of the child's reality, you've got things out of place. There's no reason, in the everyday life of most of our children of eight, nine, and ten, to learn algebra, for example. For many high school kids algebra is an unreal subject. And by that I don't mean that it's abstract and symbolic. I mean it's unreal. Certainly mathematics is a very real thing if you know what you're doing and you're interested in it and it has meaning to you. But algebra as a series of algorithmic symbols, or symbols that fit into algorithms that you can never use or apply, is unreal. So we build a schizophrenic world for kids. We literally make them schizophrenics. And then we wonder why they have no interest in education.

O'BRIEN: I think that there are two things that you are saying. One is that often what we do in schools is out of context with the everyday life of the child. And the second thing I think that you're saying--and correct me if I'm misstating--is that we often have as our goal the preparation of the child for some future world and our predictions may not in fact be carried out.

SMOCK: That's right. And I think your second point is one that often goes unrecognized. In a sense all that we do, that is real and meaningful to us, could be preparation for another situation in life, another experience, another challenge. I would accept that as a reason for always challenging the child to learn new things: to try to pose to him the possibilities that he might face, in terms of a pretty reasonable idea of what he might run into. But let's take a silly example: To try to predict the specific algorithms a child is going to need fifty years from now or a specific skill he's going to need to adapt to a particular situation fifty years from now is silly. After all, look at what we have done in adapting during the past fifty years. This is 1974. Think back to 1924. Look at all the technological things that have happened, that have drastically changed our world. Some of those we have adapted to very well, and some we have adapted to poorly; we haven't taken advantage of them in terms of a quality of life in them. T. V. is an example. Here is one clear-cut unintelligent application of the notion of free enterprise. You know: let it go, let it hang out, and look where it hangs out. If you actually watch T. V., you probably (if you are really interested in the quality of life,) watch a couple of hours a week at most and even then you are probably disturbed by distorted news reports or by unanalytic approaches to controversial issues and so on. So I think one of our major problems is, "How does schooling and education produce a group of adults who can handle present technology in terms of maintaining a quality of life and yet utilize that technology to improve the quality of life?" I don't believe you do it by making sure that they know a certain amount of mathematics or even that they all read at a particular level. I think that all education--all schooling--should involve a much larger philosophical component than it does; even for little kids.

I was very impressed by a recent experiment that actually put philosophy into the elementary grades. I'm sorry I can't recall the names. I was thunderstruck that anyone would really try such a thing but I am not surprised that they're finding tremendous impact in teaching basic philosophical notions to kids in the first six grades of school. One example I remember is of current interest, I think. One boy came in and said, "All Puerto Ricans are dirty." Then he said, "Oh, I'm sorry! Most Puerto Ricans are dirty. No, I'm sorry! The Puerto Ricans I know are dirty." Now whether that's a factual statement or not is beside the point. The fact that he checked his thought processes by philosophical criticisms and techniques is terribly important. He learned not to speak in broad generalities but only what he knew first hand. This, I think, is a big step forward.

O'BRIEN: So as you look at education you are suggesting that people concerned with education should address not "How to do it" sorts of questions, but rather, first, they should ask "What is the gist, what is the purpose of this enterprise?" Is that right?

SMOCK: The purpose and the process and how they match what I call meaningfulness is essential. I think sometimes psychologists and educators get caught in the bind of giving up words that have very precise and very understandable meaning. It's hard to define meaningfulness in symbols or in numbers. But we know what it means when we say that something is meaningful to a child and we have behavioral ways of finding out whether it is meaningful or not; we can check it against his non-school environment; we can check it against his involvement and persistence in that activity; we can check it against how he transforms whatever we give him into things that are useful to him. So

I think that's the first point.

I think the second point is that, for both teacher and child, in a different way, the philosophical (or what I call the epistemological) issues are fundamental. Now epistemology has to do with the theory of knowledge. And this has been primarily a very abstract, obscure field as part of philosophy. One of Piaget's suggestions, which educators mostly ignore, is that we've all got to become better epistemologists. We've got to know what knowledge is and how it comes about, before we know how to educate our kids. So I think teacher training ought to begin with epistemological studies.

O'BRIEN: Okay, let me hit you with the zinger. Here's the real zinger. (Okay) What for you, then, after years of study--what is knowledge?

SMOCK: Okay. That is a zinger! And I have no easy answer. The answer I have, however, grows out of my study of epistemology, primarily Piaget's epistemology. Piaget is a constructivist. And I've become what is called a radical constructivist which I think he is also. A constructivist in a philosophical sense is one who says that all knowledge--all knowledge-- is constructed by the individual. Piaget says that knowledge is an invariant, any invariant under transformation of organism-environment relations. For example, object permanence. You know the studies of early development of the permanence of an object. The very young infant sees a cup but he has no way of storing in a structured way the sense impressions that that cup gives rise to. He can't hold on to it. So the object comes and goes; when there's no sense impression, there's no cup. Now as he coordinates his sensory impressions, he structures them--hand, eye; touching, feeling, seeing, reaching, using proprioceptive data. He coordinates some of these sense impressions under continual exposure. So, you see, it's not just due to what's out there. It's what's going on inside, too. A lot of things are already pre-programmed due to hereditary and maturational processes. So he finally reaches a point where he can store that concept and hold on to it. Once he does that, then he begins to think of the cup as being out there. But what he really knows is his image of the cup because all he has is what his sense impressions told him.

Now sense impressions are very strange things. As far as we know, a neuron either fires or it doesn't fire, just like a computer. So when something hits the retina a neuron fires or it doesn't fire or several neurons fire or they don't fire. So there's no way in the world that the eyeball or the mind can reproduce what happens. There has to be structuring inside the organism. There has to be a program, if you will, of operations that put all this together and structure it. So when I respond to this cup (holds up cup) and I say, "I know there's a cup," first I know there's a cup because I can imagine a cup in my hand. I can generate a picture of it from past structures in my head and then I can test that hypothesis by looking and feeling and trying to put coffee in it. But basically every time I've got to test my hypothesis. I'm not saying there is no environment out there, that there are no objects out there. What I'm saying is that whatever is out there, there is no way in the world that I can, as an individual, establish the validity of those external things as existing. Now, four of us may agree that they exist, but I've known groups of crazy people that agreed on very crazy things. So alternately you've got to introduce not only a consensus of our constructions for validity--not only that we agree-but also the question: are they useful? Do they help us survive in the most general way?

What does this mean for education? What we're thinking about can get very abstract and it can get into deep philosophical issues, but we're interested in the education part; in the development of children, in the development of their skills and their thought processes. Basically, what it means for education is that each and every individual has to construct his own knowledge. All a teacher can do--a parent, or a teacher in the formal sense--is to provide opportunities for the child to be exposed to an environment in which there are alternative ways of constructing things, and hope the child will construct knowledge that society has found to be most useful both for individuals and for the society.

Sometimes kids are very "stubborn" if you leave them free. I think you mentioned a situation, Tom, involving a researcher interested in the algorithms that children produce for solving certain mathematical problems. What he's finding is that they come up with the craziest algorithms, crazy from a mathematical point of view but the darned things work; they solve the problem. My question to him is, "Well, which is the bad algorithm--this neat, simplified, elegant one that you mathematicians have been working with all these years or these complex, crazy ones that kids are using?" How do you tell me which is the bad algorithm? Now that's one big question. Maybe these "bad" algorithms might lead to some new mathematical systems if they really are explored, because that's the way mathematical systems are invented. Systems are not out there pure and abstract and divorced from human thinking. They're produced out of the conflicts and contradictions and the reaction to novelties by individual minds. They aren't agreed upon ahead of time. That's not how mathematical systems are invented.

Another aspect of this is that we need to know how well children who invent (and use the so called clumsy algorithms) do later on in mathematics? It may be that allowing them to invent their own ways of solving problems makes better mathematicians and mathematical problem solvers out of them than giving them these "right" ways of solving problems.

O'BRIEN: I hear you have conducted some ethological research yourself.

SMOCK: You're right.

O'BRIEN: But the sort of thing you're talking about--the philosophical position of constructionism--is very much different, I think, from what often happens in schools. Knowledge usually is regarded as some sort of stuff on the outside. i.e., bits of coal to be shoveled into a passive urn called child.

SMOCK: That's right. We have this illusion. It is not completely an illusion but there is a lot of illusion in it: anything we don't attribute to being inside us we always want to put outside. You see this a lot in political and social-political education arguments and discussions. You'll find that in quite logical arguments there is a compulsion not to use just the logic--the structure of the logical argument--but also to tie concrete details to it and so in the process a lot of misstatements of facts occur. There are a lot of unreal things that are brought into play. You see this particularly in bureaucratic systems. One of the most compelling ones I've seen grows out of my experience in the Follow-Through Program where we are concerned with enriching the educational environment of kids from poor families. It is just amazing to me to go into a city or a school district and find the logical reasons and the logical reasoning involved as to why these kids in this particular neighborhood don't learn very well. You know all the reasons that are usually given. And I say, those sound like they are pretty good arguments but why is it that you only spend $250 per capita in this district and $1250 per child in that district? What do you expect if those kids have all those disadvantages you say are immutable?

Have you ever tried to change those conditions? Of course, they have. But, you see, we get into a situation where maldistribution of educational resources is maintained for political and social reasons and not for any sound psychological or educational reasons. And I think that's one of the fundamental problems in education.

I can give you another one. I don't know if you'll want to put this on record or not. I would like to have a moratorium on all the special education funds in this nation. And I will tell you why. It distresses me to go into a school district and find behind each school building a double level laboratory with $80,000 per-laboratory reading programs with two or three Masters Degree teachers and with all the reading programs that have ever been developed. And each day they take out first, second, third, fourth, fifth, and sixth grade kids to the laboratory to teach them to read. And I walk into the classrooms and I find no working space, no library, no facilities, not even attribute blocks, or logic blocks, or anything else for the teacher to work with except old textbooks. What we are doing, it seems to me, is to focus on things "out there" with very little attention to what we should be doing: helping children to learn, providing the chance for them to construct a sensible, useful world of their own.

The issue has been raised in another way with this open classroom business. What does it mean to say "open classroom"? If you take that term literally, the door should always be open for kids to go in and out and for teachers to go in and out and for visitors to go in and out. Does it mean that kids can do what they want? There is no such thing as a school where kids can do every thing they want. They tried to blame that failure on Dewey, but he didn't say that in the first place; now they are going to try to blame it on Piaget, but he didn't say that either. There is no such thing as an unstructured learning situation. If the teacher doesn't structure it, the kid will. If you don't interest the kid, he is going to structure it in his own way and he is going to learn a heck of a lot more than you thought he learned but often not what you want him to learn. It is important to remember that every individual is always and continuously structuring his environment and his experiences in a coherent meaningful way. If there is nothing meaningful out there to attract him he is going to do it all in terms of his past experiences and his hopes for the future. And bright kids, particularly, get away with it all the time. The bright ones are the ones who are continually giving the right answers. I know a teacher who will laugh when she tells me, "I think he is daydreaming but he always has the right answer when I ask him". What concerns me is what the heck does she think he is doing between that first time when he got the answer and ten minutes later. There you have ten minutes of life that she doesn't know anything about. All she knows is that he can give the right answer, but she has no idea about his thought processes.

O'BRIEN: In the second part of our session can we talk about some of the particulars as far as you (Mathemagenics Activities Program) and Follow-Through Program is concerned? We have been talking about goals and purposes and the nature of learning in children. Could we, in the second part, come down to practical nuts and bolts issues?

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O'BRIEN: In the first part of the discussion, Professor Smock talked about the goals, the purposes of education, the nature of knowledge, and a constructivist point of view with respect to knowledge in humans and particularly in children. As a follow up to that very general first part we are going to ask him now to describe some of the particulars of the two programs with which he is associated at the University of Georgia, the Mathemagenics Activities Program and the Follow-Through Program. Would you like to start, Charles?

SMOCK: O.K. I am afraid I was a little bit wordy before, so feel free to interrupt me. The Follow-Through Program is a program that was developed by the National Bureau of the Office of Education as the follow up for Head Start kids. They quickly discovered that a summer school experience or a year in Head Start didn't turn these kids into geniuses. They still had trouble learning to read in the first grade and so forth. The first year they planned it strictly as a Head Start-like program in school. They only had $15,000,000 and they found out that that was peanuts. That went nowhere. They finally were able to structure the Follow-Through Program into a research and development program. Actually, it was to be a research, development, and application program because service to the communities was an integral part of the R & D program.

What they did was to say two things: 1) Follow-Through will be a planned variation program. We will look around the country and find educators--or people who say they are educators--who have a particular model for early childhood education and we will compare one model against another with all kinds of kids and in all kids of regions. So it's a planned variation experiment. Then, certain communities were identified as a high density, low income population community with a low tax base, which needed help. 2) The second notion that is extremely important is that the sponsor is an external change agent.

That is, the community had to buy the program of some sponsor. They had to utilize some model program. They had no choice if they wanted the Federal money. So, in a sense, it is a three way partnership. The Federal Bureau provides the dollars and some guidelines, the communities provide certain resources--the teachers, the children, the needs-- and the model sponsor provides the educational program. Every year they negotiate as to what will happen next year. Right now there are some twenty two co-sponsors which can be categorized into types quite consonant with the philosophical traditions of education. First of all, you have the behavioristically oriented model. My impression is that they are academic and skill oriented. They are of the reading, writing, and arithmetic philosophy. A second group of sponsors is concerned with what you might call personal, social, and emotional development tradition, the Rousseau tradition. They are primarily concerned with enhancing the child's self-confidence, self-concept, and so forth. They aren't against learning or academic skills; it is just that their emphasis is in providing a climate where kids develop into self-respecting human beings. The third tradition has been less emphasized but it is really the one that Dewey was interested in and which Piaget also fits. This is the general child development, general intellectual development philosophy of education. As you know, Piaget says it is a matter of what you want to do. If you want to produce kids who have particular skills, then you have one type of program; if you want to produce problem solvers and creators and problems makers then you have a different kind of tradition. He definitely is on the side of educational programs that are trying to produce problem solvers and problem creators and people who can handle novelty and change.

Now the Mathemagenics Activity Program was developed originally as part of the R & D Center of Georgia on early educational stimulation. Unfortunately, the Government stopped the funding of the R & D Center and this meant the end of Follow-Through. But the national office wanted to continue the program and they looked around to find someone who would keep the Follow-Through Program going. I was a developmental psychologist, I had some knowledge of cognitive intellectual development, I once went to a one-room school, I graduated from an undergraduate teachers' college in history and mathematics, and I guess they had heard some good things about me. So they asked me to take the program and I said I would, under certain conditions. The first thing I did was to look for a name which is kind of silly. Human beings are very much tied to their language. That "sticks and stones will break my bones but words will never hurt me" is the worst thing that anyone ever said because, in terms of social reality, words are more real than reality to most people. So I chose the name Mathemagenics Activities Program. Mathemagenics: "mathema" is learning, "genic" is origin of, and then we added the word activities. And Program means that there is a coherent set of principles that will hold together. What we wanted to do was to develop a set of activities that would promote learning. It is content-free. We don't care what age level, we don't care what content is involved, we don't care whether it is a child as a system, or a classroom as a system, or a teacher as a system, or an educational establishment. The principles of learning and change that MAP proposes, apply to all. The principles are very simple and they are not that unique, if you know your educational history, although they are tied to Piaget's theory of cognitive development in one way or another.

The first principle is that of mismatch. That says that you must have an optimum degree of discrepancy between what the person already knows and what you present to him as a challenge; which is another way of saying, it must be a meaningful task to the individual. (That is not to say, that people can't learn things that are quite distant from what they are interested in or what is meaningful to them. I have a standing offer for $20,000 in one year that I will raise the reading achievement on any standardized test by at least two years and I have never been in a classroom in my life but I will do it for you. That is a standing offer to any school system in the country. They get upset when I say, "I am going to teach the test". But that is what I am going to do. And, hell, that is all that is done in most of these programs that raise scores on standardized tests. Why we perpetuate this illusion, I don't know. Well, that's another issue, but if I keep my mouth shut, I'll make $20,000 some day. So the first thing is mismatch--that's the M in MAP. We abbreviate our program by saying it's a map program--we are mapping out the kid's developmental life.

The second is activity. All learning involves action. All learning is based upon activity. It may be practical activity that you can see and observe (what we call behavior), or it may be mental or perceptual activity. All learning is based on activity. That is another Piagetian notion. Piaget's theory of intelligence is an operational theory of intelligence. By operation he means action. So that is the A in our program.

The third part is what we call P, personal regulation or self regulation of learning. Now that is one that is easily misunderstood. That does not mean the child comes to school and does whatever he wants to do. If he did what he wanted to do most of the time, he wouldn't even come to school. So we don't fool the kid. We tell him, "You're here to learn; - We're here to help you learn. So let's not have any of this nonsense that self regulation means license or complete freedom, whatever that is. What we mean by self regulation is that within the context of an activity--sometimes it is concrete material, sometimes verbal material, sometimes it is just thinking, sometimes it is watching movies--we want these kids to learn by doing. That is what we mean by self regulation. For any group of kids there has to be social interaction and there have to be options available to the child. There are two kinds of options. One option that is practical for almost all classrooms is in terms of the level of conceptual difficulty of a task; so if you have eight kids sitting around this table, there may be three or four ability levels among those kids. And if they have a common task to do, it ought to be structured in a way so each child can learn at his particular level, with the kind of concrete materials support or verbal support that he needs. When we walk into one of our classrooms, we want to see options in terms of the kinds of ways that kids are permitted to learn a concept. Another thing in terms of options (related to self regulation) is that every child ought to have some time during the week, when he or she can spend a good amount of time--like an half hour or an hour or more--on exactly what they want to learn. So they can select their own learning activity.

This has a lot of implications, but, regarding what we mean by self regulation, there are two aspects. One is that learning is always self regulating; it is a matter of whether or not it takes. So if you have a single concept or a single task to learn, you can attract in a meaningful way all of your children of different ability levels by providing optional ways of learning that task. The other part has a much longer-range aspect: we say to kids--"we respect your own interests and you ought to learn to respect your own powers of thought." So those are the three basic principles; mismatch, activities, and personal regulation of learning.

O'BRIEN: What generalizability does this have away from your particular program?

SMOCK: Well, it has a lot. In fact we see a lot of good classrooms. I haven't been to Britain recently, but I would guess that if we saw a good Piagetian school in Britain we would see this kind of classroom. We see a lot of this kind of classroom among those called open classrooms. Dave Weikart has one. The Florida School of Education has a number of classrooms that look very much like this.

Follow-Through has gone on for seven years. The first three years we were basically a traditional subject oriented program. Now this has bothered a lot of people. But not being an educator and having gone through many years of psychological theory that was non-meaningful and research that was non-meaningful, I made up my mind that what really counted are the operations on the environment; this is what makes a difference. Now you tell me what you would do, if you walked into a community that had a series of over crowded, dirty classrooms with seats tied down, with no resources, with probably four out of the six hours being recess time, because the teacher can't control the kids. Tell me what you would do? I was not going in there with a process-oriented curriculum that talks about Piaget. What I did was very simple.

(I think I mentioned this earlier: there have only been two really significant revolutions in American education so far as I'm concerned. Number one was those things that contributed to unscrewing the desk from the floor. Education is not gained by sitting on your can. Second--and I think this is one of the things that many of use have done in Follow-Through--we opened the classroom door and took the closed classroom door as irrelevant to academic freedom. A closed classroom door is antithetical to academic freedom. Any educator who will not open his door is not a professional. He is scared and there is a number of adjectives I could apply. But if education means anything, it means that it is open in the sense that anybody at anytime can walk in.) So anyhow, what did we do? We unscrewed the desk from the floor, we opened the classroom door, we bought supplies and materials, and we gave short term workshops to the teachers, not to teach them anything but to find out what they knew. After we found out what they knew, we provided them with supplies and materials and two aides which we trained to force them to use small group instruction. We made them buy tables, so that kids would have to sit around tables - mundane things like that. Every month, we had a visitor go to the community and visit the classroom. Every other month, we had a subject matter consultant sent in to help the teacher utilize the Cuisenaire rods, the logic blocks, the ESS and AAAS science programs, and so forth. We have never said, "You must use this or that textbook or this or that set of materials". (Teachers can make their own, as far as we are concerned.) But, most of them worship these materials and textbooks and we try to wean them away from that. Now it took a good two years of this and, you know, the achievement of the kids on standardized tests went up, especially the K & 1st grade, during those two years. But what the heck did that mean? They had come from nowhere. Although their progress was great they were still below the national norms. Still, we have kindergarten kids reading. And since many of them are black kids, that kind of frightened the community, because they knew these kids couldn't learn to read - because they never had learned to read. And we have some mountain communities also, where some poor whites never learn to read, but they do now. So it is not black or white. These prejudices--it doesn't make any difference what the color is--they are there.

What we did, was to stage our teacher training according to the same principles that we apply to kids. You can't teach what you don't know. This is why I am so concerned that Piagetian theory, as applied to education, will be called a failure ten years from now. Many people don't know what the heck they are talking about with respect to Piaget and there is nothing as distressing to me, as to find a teacher training program or an early childhood program where they tell the teachers, "Now if you learn about all these conservation tests, you will know Piaget". That is distressing. It is distressing to Piaget, by the way. That also is the experience Dewey had. No one studies Piaget anymore. They are studying Elkind or Flavell, or somebody else. I just told a group of graduate students the other day, "If you get off this second and third generation bug and get back to the source, you wouldn't make so many many mistakes." And I gave them a task to do which will be very meaningful to them. You can do it in several different ways but the most meaningful one is to take the basic Piagetian concepts and go through the secondary sources and pull out the definitions that other people give to Piagetian concepts. What you will find, is that about half of them know what they are doing. Any many of those have taken direct quotes from Piaget and therefore they don't make a mistake.

But I can give you innumerable examples of mis-definitions of concepts that mislead people.

O'BRIEN: Let's go back to your classrooms. What did you hope to do with kids and do you think you succeeded?

SMOCK: We haven't succeeded completely. First we have to define success or failure and we have to recognize that any program is going to have multiple outcomes. That's one of the big failures of the educational evaluator; the notion that one program has one outcome, and that's it. Every program has multiple outcomes. The problem is to decide which of several important outcomes you want to emphasize. For Example, let's take kindergarten and first grade. If you want kids to be able to read by the end of first grade, at their level, there are ways of teaching them how to read. In fact most children with a minimum of help--no matter what your method is--will learn to read. They learn in spite of their school's structure instead of because of it. But there are lots of ways. One of the most successful programs has recently been commercially produced by Distar--The Bereiter-Engleman type of program--and this is a very successful way of teaching most kids how to read. The problem is that you say, "That's good." It is good because kids can read. But what else do kids learn in school besides reading? The learn self confidence; they learn about the source of knowledge. What is the source of knowledge; where is knowledge? They learn about the source or the locus of control of their behavior. They learn about regulation of their own behavior. So you look at the end of the second year and you ask how these kids are doing on self-concept. How about their orientation to problem-solving, i.e., do they like problems or not? How about their interest in school, their locus of control? So these kids are still at the early five year level. Whereas in Bank Street, which has a personal-social development orientation, the kids are very high in self-concept, they realize that they are partially responsible for whatever happens, and they have a lot of confidence. If they fail they tend to share the blame with the teachers, whereas if--and this is rather interesting--if the Distar program kids fail they blame it on themselves but if they succeed they blame it on the teacher. There is no mutuality at all, no recognition of the relativity of these things.

The main thing I want to emphasize is the differential outcome in terms of personal-social development and academic achievement. The question is, which would you like or, perhaps you would like to have a mixture of both. As some of us think, it's not so bad that the kid, at the end of the second grade, is not reading at the late third grade level, if he is reading at the middle of the second grade level and he has a lot of confidence in himself and can do a lot of other things besides reading. It is a bid distressing to find four or five hours, out of a six hour day, spent on reading skills; that there are in fact, other kinds of things besides reading scores to consider when you talk about success or failure.

O'BRIEN: Which brings us back to where we started, which is goals and purposes. It is important that that must be considered.

SMOCK: Right. I think in our six communities we have about 85 or 90 teachers. This is first, second, and third grade with a few kindergarten classes. Counting the teachers who have been in our program for at least three years (we have teacher turnover and that kind of thing) would give us about 40-50 teachers. Of these, I would say that half are good teachers from our point of view. They construct a classroom that we like to see. And, interestingly enough, the cognitive developmental level of their children is much advanced over the kids in the same community who are not in our classrooms. Their standardized achievement scores are just slightly below or on some sub-tests they go above. So we have done great things in terms of standardized achievement level. But these kids also tend to have a lot of self-confidence; they ask a lot of questions, and when they get into the fourth grade they are very independent learners. In fact--it is not a fact yet, but is an impression that we are beginning to get, and we will know at the end of this year--our kids who have graduated and are now in the fourth and fifth grade, are out-achieving the others by far on achievement tests. They are out-achieving the fourth and fifth graders who lived in the same community but did not go through our program.

O'BRIEN: Charles, if people wanted to find out more where would they write to? Where would they look?

SMOCK: I think if they wanted to find out more about Follow-Through they would write Miss Rosemary Wilson (Director of Follow-Through Program) at the Office of Education. If they want to find out more about MAP they would write to me and we would be glad to send them a set of materials.

O'BRIEN: And in addition to that there are a number of white papers and research reports that your project has produced.

SMOCK: Right.

O'BRIEN: Thanks to Charles Smock for joining us here.

SMOCK: Thank you.

Constructivism and Educational Practices

by

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Constructivism and Educational Practices

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Introduction

Until recently "constructivism" has played a minor role in both epistemology and science, though the fundamental notions were known and debated among the early Greek philosophers (von Glasersfeld, 1974). Relegated to being critics and, at times, petulant naggers, the constructivists found modern physical science setting the stage for a "return of the repressed." Mack, in 1886, recognized the tenuous base of any form of positivism in science: "We are accustomed to regarding the object as existing unconditionally, although there is _no such thing_ as unconditional existence. To extrapolate this experience beyond the limits of experience and _to assume the existence_ of 'thing in itself' has no intelligible meaning. We have become accustomed (only) to regarding an object as existing permanently" (Mack, 1970, p. 30). Einstein also recognized a form of constructivism when he said: "Science is not just a collection of laws, a catalog of unrelated facts. It is a creation of the human mind, with its _freely invented_ ideas and concepts. The only justification for our mental structures is whether and in what way our theories form a link with the world of sense impression" (Einstein, 1920). And, finally, the father of operationalism stated "when we say that we see a thing out there in space, we are exploiting correlations built, by experience and repetition, into the structure and functioning of our brains" (Bridgeman, 1961, p. 46). Thus, some early Greeks and modern physicists agree that the existence of objects depends on our personal constructive coordination of experiential data and the subsequent projection of these structures into the "outside" world.

Piaget made the analysis of this process of coordination of experiential data the foundation of his theory of knowledge and knowledge acquisition. His analysis offers substantial evidence that "object permanence" is the result of the subjects' coordinations of patterns of experiential data. In the process of his analysis, Piaget constructed both an epistemology and a theory of the development of knowledge that constitute fundamental contributions to theory construction, research and practice for psychology and education.

Piaget came upon the scientific and philosophical scene at a time (1920's) when a scientific revolution was apparent though not completely accepted. The history of the physical sciences contain many examples of such paradigm shifts (or broken images, if you will) but only later will we be able to determine whether or not Piaget has exacted such a revolution in the behavioral sciences. In any case, Piaget was apparently well suited to the task. We most often recognize his epistemology or psychology, but Piaget's methodological creativeness should not be ignored. His revival, and particular conception of, natural genetic epistemology required a bold, independent, as well as a capable mind. It is easy to forget that only twenty years ago his idea of building a theory of knowledge, let alone a cognitive psychology, on the basis of naturalistic, genetic, structural, and operational bases seemed absurd to most psychologists.

My purpose in this paper is to first, review the essentials of

Piaget's epistemology and of his cognitive developmental theory. Second, I will describe one application of his theory to education; specifically, a model for developmental change that applies to any system whether it be a child, a teacher or a community educational system. Piaget has clearly recognized the impossibility of direct application of his theory to educational practices. Two good reasons are immediately apparent: first, the complex theoretical structure has to be operationalized, a task that is impossible without construction of a "model" containing a coherent set of "essential" principles to guide practices. Second, Piaget (1971c, p. 2) warns that we must clearly differentiate pedagogy from psychology. Experimental pedagogy is concerned less with the general and spontaneous characteristics of the child than with their modification through the teaching-learning process. Moreover, Piaget rejects the motion of inflexible stages characterized by invariant age limits and fixed thought content as a necessary, or valuable, consequence of the concept of developmental stage (1971, p. 171). Cognitive structural changes that come about through maturation and those that derive from the child's individual experience can be, and often need be, considered as separate factors in intellectual development. For example, available research suggests that mathematical structures can be learned if the structures of interest are supported by more elementary structures. Thus, while the work of the Genevan group has dealt with the development of the child independent of particular subject matter curricula, experimental pedagogy is a necessary compliment of cognitive developmental psychology. Steffe's (1975) theoretical and empirical analysis of the degree of correspondence between mathematical and psychological operational structures clearly illustrates the potential of this position.

Piaget's Constructive Epistemology

Piaget's constructivist theory of knowledge and knowledge acquisition is complex and I have no intention of trying to add new insights into the many epistemological issues that have been and can be raised about that Piagetian theory. Here, the essentials of his position will be sufficient to indicate the necessity for consistency among the epistemological assumptions, the theory of cognitive development and a proposed model for educational change.

Piaget's primary assumption is one of subject-object unity. Knowledge does not arise from the object nor from the subject, but from their interactions; that is, between the subject and the perceived object. To know objects the subject must act upon, and transform them. These transformations consist of actions that connect, displace, combine, take apart, and reassemble, etc. To know an object (reality), then, means to construct systems of transformations that can be carried out with objects. Such transformational systems constitute knowledge but are not copies of reality--only possible isomorphic models of reality from which we may choose.

Knowledge acquisition is a biologically oriented process, then, in which a subject evolves his own objective sense of reality. That evolution or construction of knowledge is assumed to proceed through two types of activities: the coordination of actions and coordinations related to "objects." The structures of actions (knowledge) are constructed in that they are not in the objects (they are dependent on the action) nor is it given in the subject (the subject must generate coordination of the actions). Thus, the notion of action (or operation) and transformation are central to Piaget's constructionistic epistemology. He emphasizes over and over that knowledge is a continuous construction and in each act of understanding, some degree of invention is involved. And, in development, the transition from one stage to the next is characterized by formation of new structures which previously were neither in the external world nor in the minds of the subject.

The construction of knowledge, as indicated above, involves two types of action structure that give rise to two types of knowledge: physical and logical-mathematical. Physical knowledge is derived from activities related to the subject's abstractions from objects themselves. Generally, physical knowledge relates to the figurative aspects (Furth, 1969) of knowing because the subject attempts to represent reality as it appears without transforming it. Logical-mathematical knowledge on the other hand, is derived (abstracted) from activities and operations themselves. That is, logical-mathematical knowledge emerges from the subject's reflection on his own coordinating activity which yields an expanded internal structure of logical-mathematical relations. The notion of reflective abstraction, and the conditions for generation of logical-mathematical knowledge, is one of the revolutionary aspects of Piaget's epistemological position for education. The implication that logical-mathematical knowledge is not directly teachable--but rather consists of invention by the subject himself--has been more than disconcerting to many psychologists and educators. The tenability and/ or limits to the generalization is, as yet, empirically unknown, and in my view does require a new perspective on the teaching-learning processes. It does not prohibit, however, the development of instructional practices designed to facilitate reconstruction of existing (social) knowledge consistent with Piaget's theory of cognitive development.

The final aspect of Piaget's epistemology that educators must consider is that the processes of construction of logical-mathematical and figurative knowledge are functionally different than the symbolic processes by which the subject represents actions. An object or event within the subject-object interaction that provides some knowledge to the subject about another object or event is a "signifier." An index representation involves a direct relationship between an object and a representation of that object; e.g., a cry is an index of a baby's presence. A symbol is differentiated from an object but retains a degree of similarity to objects; e.g., a child represents a car with a block of wood. Finally, the third type of signifier is a sign which is differentiated from the significant but is conventional and often arbitrary; e.g., linguistic or other socially agreed upon representations are considered signs.

These epistemological assumptions did not appear full-blown, but were the product of both theoretical and empirical analysis over a period of years. Piaget's biological background and early commitment to a natural genetic methodology inevitably, it now seems, directed him toward an operational and structural analysis of knowledge acquisition. The operational theory of intellectual development that emerged is characterized by a complexity that does not allow for quick or easy understanding, appears inherently ambiguous at certain parts, and has an openness that permits gradual (or partial) modification without serious implication for the theory as a whole.* During the past ten years, however, the major aspects of the theory of cognitive development have become much clearer. Both psychologists and educators are utilizing Piaget's conceptualizations for research and for practice (an mal-practice.). In the next section, I will discuss what, in my opinion, is the minimum set of assumptions in the operational theory of intelligence necessary to generate models for educational practices. The reader should be cautioned that consistency among the epistemology, cognitive developmental theory, and "model" derived from the theory must be evaluated, but it is the theory, and not the epistemology, that is of primary concern to the research psychologist and educator.

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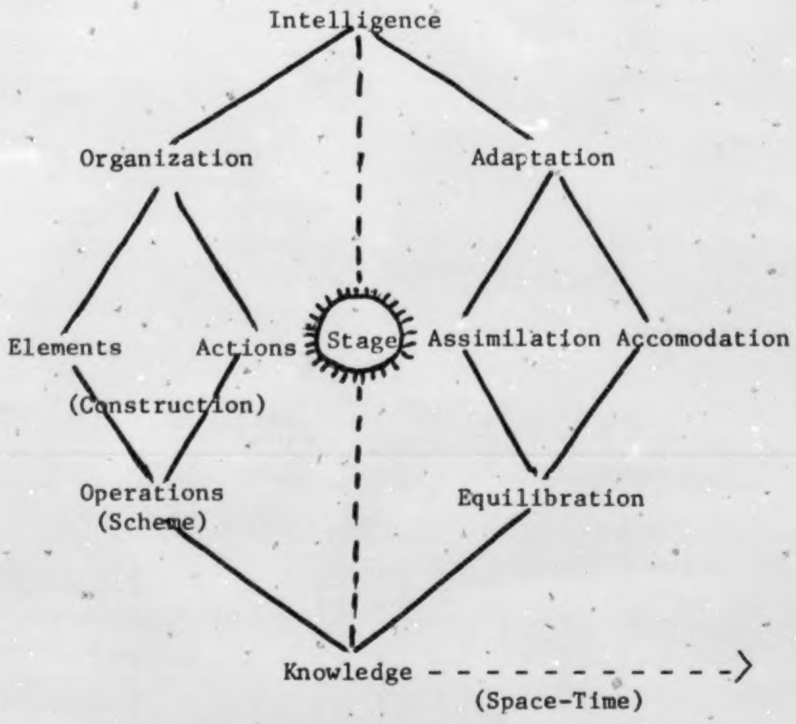
*For example, Piaget's recent recognition of the significance of advances in "category theory" as requiring analytic attention to "correspondence" as well as "transformation" in his theory of knowledge and knowledge acquisition (Abeles, 1976).

Operational Theory of Intelligence

Piaget's constructivistic epistemology, and his biological background, predisposed him toward an operational and structural analysis of intelligence or the knowledge acquisition process (Piaget, 1967; 1968; 1970a; 1970b; 1971a). The essential concepts of Piaget's theory are schematically represented in Figure 1. Knowledge is defined as an

Figure 1

Core Concepts of Piaget's Theory of Cognitive Development


Intelligence
Organization
Adaptation
Elements
Actions
Stage
Assimilation
Accomodation
(Construction)
Operations
(Scheme)
Equilibration
Knowledge
(Space-Time)

invariance under transformation (a most familiar concept to mathematicians) whereas intelligence is the activities of construction of invariances in organism-environment relations. Thus, the two aspects of intelligence (adaptation and organization) eventuate in a stable or equilibrated scheme(operational structure) or "knowledge." An understanding of both the structural (organization) and process (adaptation) components of the theory, as well as the complimentary and reciprocal nature of their relationship, is necessary in the development of adequate models for educational practices at all levels.

Adaptation consists of the operation of two complimentary biological processes and the internal self-regulating mechanism of equilibration. One of the two processes (assimilation) concerns the application of cognitive operational systems (structures) to the organization of sensory data. New data or events are incorporated into existing structures through both on-going physical and mental activity. Such events and the products of new experience can be incorporated into the cognitive structure only to the extent they are consistent with existing functional structures. Accommodation is the complimentary process whereby adaptation occurs by integration of existing structures with other functional structures and/or by differentiation of new structures under confrontation with new experience.

Activities such as play, practical or symbolic, represent assimilative activity, whereas memory, in the sense of involving past experience, and imitation are accommodative since only prior formed structures are transformed for a new use or application. Assimilation is an active constructive process by which the data from experience are transformed and integrated with an already generalized cognitive structure. Accommodative activity, on the other hand, is the process whereby modified existing structures or novel structures are brought to bear on newly assimilated sensory data.

The second aspect of intelligence concerns the organization of sets of mental operations necessary for maintaining invariance under transformation (i.e., knowledge acquisition). Piaget recently (1970b) elaborated his position that all human beings possess the same biological structures and functions that, in "exchange" with the common features (elements) of the natural world and mental operations generate structures (scheme) supporting the behavioral functions characteristic of each stage of development. Logical thought, in the Piagetian sense, is universal and of fundamental importance to an understanding of development and learning. But, whereas Chomsky maintains the human mind is "programmed" at birth with cognitive structures (i.e., mental representation of a universal grammar), Piaget accounts for the universality and stability of structures across cultures (Piaget, 1967) in terms of the self-regulating mechanism of equilibration. Thus, the mind at any point in development is the unfinished product of continual self-construction (Piaget, 1971a). "Logical" processes are generative and not fixed. Structures are not performed, but self-regulatory, transformational systems with the functional factors in that construction being the processes of assimilation and accommodation. It follows from these considerations that there is an "inherent logic" to development.

Operational systems consist of elements and laws of combination of those elements that form a "logical" closed system. These mental structures are observable in the actions of the organism in its environment. They are describable in terms of formal or logico-mathematical structures. Genetic psychological analyses of these structures are a necessary prerequisite to an understanding of thought processes since there is "no structure without genesis, no genesis without structure."

During the sensori-motor period of development, action structures of the individual are revealed in "practical" groups; i.e., the coordinated actions of the individual (Forman, 1973). During the pre-operational period, the child constructs representations called figurative structures which do not have the operational property of reversibility. Piaget was able to identify operational structures with mathematical system properties in children between ages five and seven. The discovery of a resemblance between the structure of the mental action system (reasoning or thought) and mathematical structures (i.e., mathematical groups and lattices) has a profound effect on Piaget's thinking. Thought, it would appear, has the same or similar properties as mathematical group structures and both are governed by the same internal logic.

The basic structuralistic approach of Piaget involves finding or creating logico-mathematical systems that describe the thought processes of an individual. Mathematical group and lattice theory are algebraic systems (Piaget, 1957a, 1957b) that might well describe operational thought and Piaget has taken maximum advantage of this possibility. To review the fundamentals: a mathematical group is a system consisting of a set of elements, together with an operation (law of combination), which yields the following system properties: (1) when applied to the elements of the set, the combinatorial operation will yield only elements of the set; no elements external to the set can be produced; (2) each set contains a neutral (or identity) element that, when combined with another element of the set, yields the same element set; (3) each element of the set has an inverse belonging to the set which, in combination with any other element, yields the neutral or identity element; (4) the combinatory operation is associative; i.e., $[(n + m) + p = n + (m + p)]$. Piaget found it necessary to generate a "grouping" model with additional properties (i.e., both group and lattice properties) to describe the concrete operational structures. Most importantly, the properties of these psychological groupings are not derived from the properties of things, but from action patterns in relation to things. Thus, the elements of psychological groups are themselves transformations that characterize the individual's operations as he acts upon sense data.

The task of the developmental psychologist is to describe the nature of action structures of the child at each point in development and, as much as possible, to formalize those descriptions in terms of logico-mathematical terms. The task of the educational researcher and psychologist, on the other hand, requires attention to both the logical and empirical validity of Piaget's system and to the correspondence between the characteristics of the psycho-logic systems and those logic (conceptual) structures derived from the relevant subject matter areas (e.g., mathematic, science, etc.).

Two aspects of the theory require special consideration by educators; the role of experience in cognitive development and the distinction between operative and figurative "knowing." The choice of these two issues for discussion here is based on a view that "schooling" and "education" can be, but seldom are, synonymous. That is, the objective of instruction is to provide controlled experiences that facilitate the individual's reconstruction of a socially existent knowledge base but under conditions that promote, optimally, the development of operational thought (problem solving and reasoning).

Equilibration Model: Role of Experience

Experience is not a unique factor in development according to Piaget. Merely being exposed to particular environmental situations is conducive neither to cognitive activity nor to developmental change. Children may or may not make discoveries in the course of play or watching a laboratory experiment (or conducting one), and listening to a lecture, etc. may or may not help a child acquire a particular concept. Equilibration is the central factor in structural change whether that refers to "stage" or to "concept" learning. Equilibration is the process of intrinsic (self) regulation that balances assimilatory and accommodatory processes, compensates for external and internal disturbances (internal or external to a particular structure) and makes possible the development of more complex, hierarchically integrated operational structures. "Rhythms, regulations, and operations, these are the three essential procedures of the self-regulation and self-conservation of structures; anyone is, of course, free to see in this the "real" composition of structures, or to invert the order by considering the operative mechanisms the source of origin, in an atemporal and quasi-Platonic form, and by deriving everything from these mechanisms. In any case, however, it will be necessary, at least with regard to the building up of new structures, to distinguish two levels of regulation. On the one level, the regulation remains internal to the already formed or nearly completed structure and, thus, constitutes its self-regulation, leading to a state of equilibrium when this self-regulation is achieved. On the other level, the regulation plays a part in the building up of new structures and integrating them as substructures into larger ones." (Piaget, 1968, Ernst von Glasersfeld, translator).

Disequilibrium occurs as the child assimilates data from immediate experience into existing mental structures. As cognitive structures change to accommodate to the new experiential data, equilibrium is restored. The equilibration process is one of auto-regulation--both of the transformations of data based on existing cognitive structures and of changes through accommodation. Thus, the child must be exposed to situations that are likely to "engage" the functional structures; i.e., he must be involved in a personal striving to understand or "accept" the task as a "problem."

One basic question for instructional theory and practice is: What are the processes and conditions that motivate and insure engagement or acceptance of the problem task by the child? The source of "interest" that promotes striving for problem solution is contingent on assimilative-accommodative activity but the specifics remain unclarified in Piaget's theory (cf: Mischel, 1971). Within a structuralist framework--if a structures exists, it must function--multiple cognitive structures provide a dimension of openness that make probable continual sources of disequilibrium from interaction of the internal operational and/or figurative structures activated, as well as by exchanges involving novel experience. And, despite lack of specifications, Piaget is quite explicit on his general position: "It is not necessary for us to have recourse to separate factors of motivation in order to explain learning; not because they don't intervene . . . but because they are included from the start in the concept of assimilation . . . to say that the subject is interested in a certain result or object thus means that he assimilates it or anticipates an assimilation and to say that he needs it means that he possesses schema requiring its utilization" (Piaget, 1959).

In passing, it might be noted that natural or life-like contexts seem to provide excellent situations for promoting cognitive change. Unfortunately, too little empirical investigation has been oriented to questions of the natural environmental determinants of curiosity of children at various stages of development and with different experiential backgrounds; i.e., what children recognize as problematic, and what kinds of incongruities are sufficient to motivate change in concepts and/or beliefs.

Cognitive conflict, or the awareness of a momentary disequilibrium, generates a need to establish consistency (equilibrium) between the existing schemas and/or novel information and is motivation for cognitive activities. Both application of an existing schema, and the elaboration of new ones in the course of development, stem simply from the overriding need to make "sense" of present problems by fitting them coherently into schemas "learned" in the course of solving prior problems.

The fundamental difference between the equilibration and associationistic theories of learning (Piaget, 1957b) is the notion that disturbances introduced into the child's systems of prior formed scheme leads to the adoption of a strategy of information processing. For associationistic theories of learning, "what is learned" depends on what is given from the outside (copy theory) and the motive that facilitates learning is an inner state of some sort or other. Equilibration theory holds, however, that learning is subservient to development; i.e., what is learned depends on what the learner can take from the given by means of the cognitive structures available to him. Further, cognitive disequilibrium (functional need) is what motivates learning; i.e., questions or felt lacunae arising from attempts to apply schemas to a "given" situation. The child, then, will take interest in what generates cognitive conflict; i.e., in what is conceived as an anomaly. If the task demands are so novel as to be unassimilable or so obvious as to require no mental work, the child will not be motivated.

After the period of sensori-motor development, equilibration becomes a process of compensating for "virtual" rather than actual disturbances. At the operational level, intrusions "can be imagined and anticipated by the subject in the form of the direct operations of the system--the compensatory activities also will consist of imagining and anticipating the transformations to an inverse sense" (Piaget, 1957b). Further, there need be no external intrusions in order for the equilibration process to be activated. For example, the acquisition of conservation concepts is, in Piaget's view, "not supported by anything from the point of view of possible measurement or perception--it is enforced by logical structuring much more than by experience" (Piaget, 1957b). It is the "internal factors of coherence--the deductive activity of the subject himself" that is primary. Equilibration, then, as we noted earlier, is a response to internal conflict between conceptual and/or figurative schema rather than a direct response to the character of environmental structure factors. Bridgman (1936) expresses the same idea when he states "we do not experience things; things are a construction of ours the function of which is to emphasize the resemblance between aspects of our present immediate experience and aspects of our past experience . . . "

Equilibration Model: Operative and Figurative Processes

A considerable amount of confusion concerning Piagetian theory and its implication for both research and instructional practice derives from a failure to consider the figurative and operative aspect of intellectual functioning. In general psychological terms, the distinction is between the selection, storage and retrieval, versus the coordination and transformation of information (Inhelder *et al*., 1966). More specifically, the development of any sequence of psychological stages, *a la* Piaget, consists of an interactive process of equilibrating functional structures of the organism with sensory event-structures of the perceived environment. Piaget's (1967) analyses "experience" into the two components of "physical" and "logico-mathematical" and this distinction between a physical and logico-mathematical experience is essential to the understanding of the growth of knowledge. Knowledge based on physical experience alone is knowledge of static states of affairs; if a child is wrong, it is easy to demonstrate that to him. Knowledge emerging from logico-mathematical experience is knowledge of transformation of states and quite another matter. If a child is wrong in the first case, it is easy to demonstrate his error, but in the second it is difficult, if not impossible, to convincingly demonstrate or even to get the child to accept verbal explanation of the "correct" answer. For example, if a child fails to *align* the two endpoints when comparing lengths of sticks, it is quite easy to correct the mistake. If, however, he fails to display *transitive reasoning* in a task, one or two examples is not likely to "teach" him the concept.

Physical experiences, then, provide for the construction of the invariants (figurative process) relevant to the properties of objects (i.e., states) through exchange with objects involving the sensory mechanisms. For example, one may touch something and it is hard, cold, hot, soft, supple, etc.; or one may see something--an object is red, a diamond cutting glass, the shape of a banana, etc. The course of logico-mathematical experience, however, is assumed to be abstractions (operative knowledge) from coordination of actions vis-a-vis representations of "objects," i.e., transformations of the "states" associated with series of discreet physical experiences. The critical difference is that logico-mathematical knowledge demands that a pair (or set) of physical objects not be defined by the temporal-spatial (perceptual) similarities, but rather by invariant relations among or between objects.

Figurative and operational processes, then, represent two types of functional structures necessary to account for knowledge acquisition. Figurations are defined as those action schemes that apprehend, extract and/or reproduce aspects of the prior structured or organized physical and social environment. Such action schemes include components of perception, language, imagery, and memory. Figurations and associated acts are based on physical, as contrasted to logico-mathematical, experience and constitute the "empirical" world; i.e., empirical truth is no more than the "representation of past experience in memory."

Operations, on the other hand, do not derive from abstractions from objects and specific events; rather, operational knowledge is derived by abstractions from coordinated actions relevant to those events. Thus, operations are those action schemata that construct "logical" transformations of "stages." Such "logical" systems of transformations operate either upon representations of events, or on the cognitive system's own logical operations.

Figurative and operative structures are two paralleled streams with their genetic or developmental origins in the same source (Piaget, 1967, 1968, 1970a, 1970b; Piaget & Inhelder, 1971)-the sensori-motor structures. Logical (operational) structures are not generated from the figurative schemata, i.e., not from perception, memory, etc. Reciprocally, figurative structures do not derive from operative schemes, but from the representations of past states of events derived from physical experience. And, most importantly, figurative structures do not derive from each other, but have unique bases in sensori-motor schema. Imagery, for example, is a derivative of deferred sensori-motor imitation (Piaget, 1951, 1952; Piaget & Inhelder, 1971) and not perception.

The postulation of these quite distinct functional structures is one of the cornerstones of Piaget's theory of knowledge acquisition and cognitive development (cf: Furth, 1969). Both the source and function of the structures are theoretically distinct. Operative structures derive from abstractions from coordinated actions, figurative structures derive from sensori-motor and perceptual activity. Operative structures produce "logical" transformations (conservation of relational invariants) whereas figurative structures reproduce sensory-perceptual consequence of externalized (and, thus "environmental") configurations. The variant operative structures of the intuitive, the concrete, and the formal levels form the discontinuous sequence of stages of cognitive development. On the other hand, figurative structures are static and dependent directly upon the data of experience (sensory-perceptual consequence of stimulation). Thus, Piaget makes the fundamental assumption that all knowledge acquisition activity is constructive, but the construction of figurative representations is quite a distinct process from the constructive activity at the operative level.

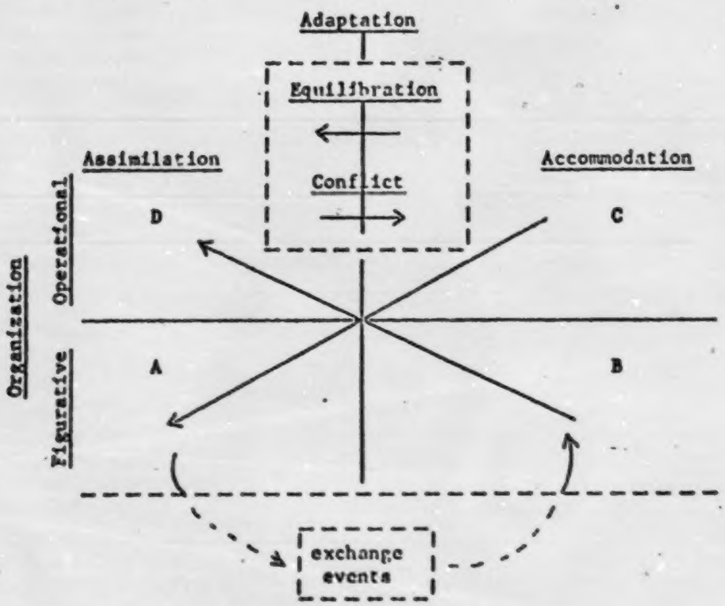
Logically, there are three possible relations between the figurative and operational structures (Langer, 1969). First, they may be unrelated and, if so, as mentally segregated functional structures, do not set limits on the functioning and development of each other. Second, psychological phenomena might be reduced to one of the types of structures. Langer (1969) suggests that subjective idealists try to reduce psychological phenomena to assimilatory operations and, on the other hand, there many who try to reduce all mental phenomena to accommodatory figurations. Naive realists, for example, propose that all knowledge acquisition processes are figurative (e.g.; perception is knowledge; Michotte, 1963; Garner, 1962). Third, and the one proposed by Piaget, is that of partial communication between figurative and operative structures within the constraints of the assimilation and accommodation processes.

The relation and the potential form of interaction are schematically presented in Figure 2 below.

Figure 2

Relations of Two Invariant Processes

of Adaptation and Two Types of Cognitive Structures


Adaptation
Equilibration
Conflict
Assimilation
Accommodation
Organization
Operational
Figurative
D
C
A
B
exchange events

Langer (1969) has examined the organizational and developmental (i.e., transitional) impact of accommodatory figurations on assimilatory operations (B, Figure 1); i.e., how does the child mentally extract and/or represent empirical information about physical and social objects and the consequences of that empirical activity for the construction of logical concepts. Imitation of an observed event, comparison of one's predictions with the perceived outcome of a physical manipulation, comparison of an observation or appearance (i.e., immediate experience) with the way things have been constructed and externalized, represent different modes of introducing internal conflict and cognitive-structural change. Generally, his findings are confirmatory, but not definitive with respect to the Piagetian hypotheses. In any case, if the development of each type of functional structure has implications for, but not direct causal effects upon, the structure and development of the other, current paradigms for the study of concept learning will require considerable modification. The work of the Geneva group concerning, for example, memory ($A = f\ [D \rightleftarrows C]$, Figure 1); Langer's (1969) analysis of the impact of accommodatory figurations (i.e., imitation, etc.) on assimilatory operations ($D = f\ [B \neq A]$); and more recent work (e.g., Bruce, 1971; Kidder, 1976; Leskow & Smock, 1970; Meyer & Elkind, 1975) all represent beginnings in this direction.

The research paradigms involved in such theoretically relevant studies are not the (apparently) straightforward ones common to the behavioristic traditions. But, more and more research reports are indicative that increased knowledge of the complexity of Piaget's theory can stimulate ingenuity and inventiveness of both the psychological and educational researcher. As we argued, many years ago, experimental and operational "simplicity" are two quite different things. The fact that we are beginning to face the complexity of psychological developmental processes (even discontinuity) is partially due to the power and viability of Piaget's theoretical constructions.*

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*Psychologists and educators alike have been slow to recognize that a constructivist epistemological base such as Piaget's leads to the assumption that the environment, essentially, is a block box (von Glasersfeld, 1974) and must be "defined" by the subject. Thus, the stages of psychological development should have their counterparts in terms of "layers" of the environment. Moos (1973) and Gump (1976) provide a starting point for such analyses.

Implications for Instructional Practices

The complexity of Piaget's theory is such that a variety of models for educational practice have been proposed. The implication of Piaget's epistemology (DeVries and Kamii, 1976), his theory of representational processes (Sigel, 1971), and the stage-dependent cognitive skills (Weikart, 1971) are a few examples of the diversity of "essentials" chosen as necessary for education. While in some cases the differential emphasis is legitimate (e.g., Piaget views the representational processes as functionally separate); in other cases there is incomplete modeling of the operational (in the practical sense) implications. In fact, there seems to be a growing opinion that the stage dependent processes and the implications thereof can be ignored in educational practice (Brainerd, 1978). Space does not permit full discussion of that issue here, but it will become obvious that I disagree strongly with that position.

The model for instructional practices, and for educational system change generally, described below, was developed in the context of the Follow Through Program (Smock, 1969, 1974), a nationwide research and development effort to improve the educational status of children from low-income families in the public schools. The model principles and specifications were derived from Piaget's equilibration model for knowledge acquisition with some special considerations of the stage dependent theory of cognitive development during the early school years.

Applications of the theoretical propositions to educational (teacher-learning) practices necessarily involve generation of a set of principles (i.e., a model) to provide criteria and specifications for teacher strategies and behaviors. The principles chosen were as follows:

First, cognitive capacities determine the effectiveness of training. For example, ability to solve class inclusion problems implies that the child already has the requisite single and multiple classification operational system for classes (i.e., combination, reversibility, etc.) in addition to appropriate information selection, storage, and retrieval abilities.

Second, the structure of learning environment must be considered relative to two frames of reference; that is, in terms of the operational systems controlling the child's interpretation of "environmental" events and the "content" to be learned. Operational systems are expressed behaviorally in the coordinated actions of the child confronted with changes in his physical and social world. For example, the mental operations of associativity, reversibility, etc., are inferred from the manner in which the child attempts to solve problems involving regular environmental contingencies (causality), understanding of spatial relations, etc. Substantive areas, such as science or mathematics, also must be analyzed in terms of their own logical sequence and structure as well as commonalities with other content areas. Certain concepts in the physical science, language, and mathematics, for example, have an inherent sequence and structure; i.e., certain concepts are necessary precursors to other, higher order, concepts. Optimal educational conditions require, then, thorough understanding of the psychological-cognitive capacities of the child as well as the sequential structuring of concepts within a particular curriculum area.

Third, optimal conditions for structural reorganization, learning in the broad sense, require disequilibration (between different conceptual systems and/or between intrinsic functional structures and the patterning of environmental events). This condition is met when there is an appropriate "mismatch" between the cognitive capacities of the child and the conceptual demand level of the learning task. Too little or too much "pressure" may result in over-assimilation or over-accommodation respectively and not be facilitative of cognitive-developmental change.

Fourth, facilitation of learning requires analysis of two levels of cognitive functioning-figurative and operative processes. The operational theory of intellectual development (i.e., Piaget) does not deny the value of "provoked" learning (i.e., through imitation or algorithms), but considers such learning to be of limited generalizability and to have minimal effects on the basic intellectual processes.

Fifth, the nature and variety of the child's "exchanges" with the environment must be considered in educational planning. The nature of the interaction refers to the relative emphasis on ontogenesis (self-directed), as contrasted to exogenesis (environmental or teacher-directed), structure of the learning environment. The variety of interaction refers to different types of structured curriculum contents relevant to the child's physical, social, and symbolic experiences. The interlocking nature of substantive curriculum areas makes it possible to provide such a variety of experiences relevant to acquisition of the cognitive "products" that provide representation of the environment such as memories and/or symbol system and, at the same time, to facilitate the development of coordinated rule systems associated with cognitive operational development. For example, analysis of the visual environment (attention or observation skills) as well as cognitive operational structures (e.g., conservation of area) can be emphasized in science, social studies, mathematics and art.

The implementation of these conditions in the classroom requires: 1) Specification of teaching strategies and tactics and 2) Sequentially structured curricula paralleling, within the limits of our knowledge, both the concept hierarchies of a particular "content" domain and the psychological capabilities of the child. In the case of teacher strategy we provide the teacher with both theoretical background and, more importantly, we find specific demonstrations and classroom experiences designed to increase their competencies to implement the following principles in a variety of contexts:

1) Since the source of motivation for change is provided by a discrepancy (disequilibration between different conceptual systems and/or between previously acquired conceptual system and environmental task demands) an appropriate mismatch (M) is necessary to generate exploratory activities and insure the individual has the prerequisite conceptual basis for learning higher order concepts.

2) Since coordinated actions are the bases for knowledge acquisition, the learning environment must be structured so that specific task demands include appropriate practical, perceptual, and mental activity (A).

3) Knowledge acquisition involves construction of invariants both from properties of objects (physical experience) and from the child's actions objects (logico-mathematical experiences). The learning environment must include provisions for personal or self-regulatory (P) constructions. Practically, self-regulation implies a variety of options, in terms of task difficulty, mode of learning,

and choice of activity, as necessary ingredients of developmental change and learning.

Implementing these principles (MAP) is not an easy task for a teacher. A supportive physical and social environment (i.e., learning centers) helps, but the first major task involves the teacher acquiring a "Piagetian perspective" of children and their development. Secondly, curricula materials and guidelines must be developed that "match" the teaching strategy or process. The application of Piaget's theory to education will, in my opinion, not be generally successful until such a match is achieved. The stage dependent aspect of Piaget's theory that has been given the most attention is that of generally "matching" the conceptual level of tasks to the current operative level of the child. However, curriculum "contents" based on the correspondence between the hierarchical concept structure of a knowledge domain and the psycho-logic (operational and figurative) structures of the child is a necessary condition for educational practices. Until that profound issue is settled (area by area), the more immediate concern is the matter of facilitating the reconstruction of the various knowledge bases in the context of both the stage independent process and any constraints on learning that are inherent in the stage dependent cognitive operational systems. These constraints, I maintain, should not be ignored, but rather determined in the context of the concepts to be learned. Specifically, the teacher needs to have a procedure for determining "appropriate mismatches" and the extent to which the child's learning is dependent on figurative and/or operative processes.

One approach to providing such information has been suggested by Steffe and Smock, (1975, 1976) for mathematical instruction.

Specifically, it is assumed that the process of concept development involves discontinuity similar to that noted at the macro-development level (i.e., stages). Instructions for concept learning, then, should begin with provision for exploratory activities (phase I) before introducing tasks designed to provide opportunities for abstraction and representation (phase II), and, finally, formalization (phase III). Whether this first approximation of concept learning phases proves valid or not, it provided a strategy for helping children reconstruct available knowledge under conditions that recognize both the discontinuity of the learning process and the fundamental nature of all knowledge acquisition, that is, knowledge is always a personal construction.

A final note-the variety and richness of the many models for educational programs derived from Piaget's theory are, themselves, testimonies of the constructive nature of knowledge as well as its source within each of our minds. Thus, each of us has made choices in a garden (Ceccato, 1974) rich in possible decisions and my flowers may well be your weeds. But, we should keep in mind that Piaget's theory was constructed by Piaget and it is only at that level he would want any disciples.

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